MTH 1126 - Test #2 - Solutions

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Pat Rossi　　　　　　　　　　　　Name ____________________

Instructions. Show clearly how you arrive at your answers.

1. $\frac{d}{dx}\left[\underbrace{e^{\tan\left(4x^2\right)}}_{e^u}\right] = \underbrace{e^{\tan\left(4x^2\right)}}_{e^u} \cdot \underbrace{\sec^2\left(4x^2\right)\cdot 8x}_{\frac{du}{dx}} = 8x\sec^2\left(4x^2\right)e^{\tan\left(4x^2\right)}$

2. $\frac{d}{dx}\left[\ln\sqrt{4x^2-7x}\right] = \frac{d}{dx}\left[\ln\left[\left(4x^2-7x\right)^{\frac{1}{2}}\right]\right] = \frac{d}{dx}\left[\frac{1}{2}\cdot\ln\left[\left(4x^2-7x\right)\right]\right] = \frac{1}{2}\cdot\frac{d}{dx}\left[\underbrace{\ln\left(4x^2-7x\right)}_{\ln(u)}\right] =$

   $\frac{1}{2}\cdot\underbrace{\frac{1}{4x^2-7x}}_{\frac{1}{u}}\cdot\underbrace{(8x-7)}_{\frac{du}{dx}} = \frac{8x-7}{8x^2-14x}$

3. Given that $\ln 2 \approx 0.7$ and $\ln 3 \approx 1.1$, Approximate the following, using properties of logarithms

   (a) $\ln\left(\frac{2}{81}\right) = \ln(2) - \ln(81) = \ln(2) - \ln\left(3^4\right) = \ln(2) - 4\ln(3) \approx 0.7 - 4(1.1) = -3.7$

   (b) $\ln(36) = \ln\left(2^2\cdot 3^2\right) = \ln\left(2^2\right) + \ln\left(3^2\right) = 2\ln(2) + 2\ln(3) \approx 2(0.7) + 2(1.1) = 3.6$

4. $\int \frac{2x+3}{3x^2+9x}dx = \int \underbrace{\frac{1}{3x^2+9x}}_{\frac{1}{u}}\underbrace{(2x+3)\,dx}_{\frac{1}{3}du} = \int \frac{1}{u}\left(\frac{1}{3}du\right) = \frac{1}{3}\int\frac{1}{u}du = \frac{1}{3}\ln|u+C|$

   $= \frac{1}{3}\ln\left|3x^2+9x\right| + C$

	u	$=$	$3x^2+9x$
$\Rightarrow$	$\frac{du}{dx}$	$=$	$6x+9$
$\Rightarrow$	du	$=$	$(6x+9)\,dx$
$\Rightarrow$	$\frac{1}{3}du$	$=$	$(2x+3)\,dx$

5. $\int \underbrace{e^{\sec(4x)}}_{e^u} \underbrace{\sec(4x)\tan(4x)\,dx}_{\frac{1}{4}du} = \int e^u \frac{1}{4}du = \frac{1}{4}\int e^u du = \frac{1}{4}e^u + C = \frac{1}{4}e^{\sec(4x)} + C$

$$\boxed{\begin{array}{rrcl} & u & = & \sec(4x) \\ \Rightarrow & \frac{du}{dx} & = & 4\sec(4x)\tan(4x) \\ \Rightarrow & du & = & 4\sec(4x)\tan(4x)\,dx \\ \Rightarrow & \frac{1}{4}du & = & \sec(4x)\tan(4x)\,dx \end{array}}$$

6. $\frac{d}{dx}\left[\underbrace{\cos^{-1}(3x)}_{\cos^{-1}(u)}\right] = \underbrace{-\frac{1}{\sqrt{1-(3x)^2}}}_{-\frac{1}{\sqrt{1-u^2}}} \cdot \underbrace{3}_{\frac{du}{dx}} = -\frac{3}{\sqrt{1-9x^2}}$

7. $\int \frac{e^x}{5+e^{2x}}dx = \int \underbrace{\frac{1}{\left(\sqrt{5}\right)^2 + (e^x)^2}}_{\frac{1}{a^2+u^2}} \underbrace{e^x dx}_{du} = \int \frac{1}{a^2+u^2}du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C = \frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{e^x}{\sqrt{5}}\right) + C$

$$\boxed{\begin{array}{rrcl} & a^2 & = & 5 \\ \Rightarrow & a & = & \sqrt{5} \\ \Rightarrow & u^2 & = & e^{2x} \\ \Rightarrow & u & = & e^x \\ \Rightarrow & \frac{du}{dx} & = & e^x \\ \Rightarrow & du & = & e^x dx \end{array}}$$

8. $\int \sin^3(x)\cos^4(x)\,dx =$

   1. Reserve a factor of $\sin(x)$ as our future du.

      $= \int \sin^2(x)\cos^4(x) \underbrace{\sin(x)\,dx}_{\text{future } du}$

      Note: We intend to let $u = \cos(x)$

   2. Re-write the remaining sines in terms of $\sin^2(x)$

      Done.

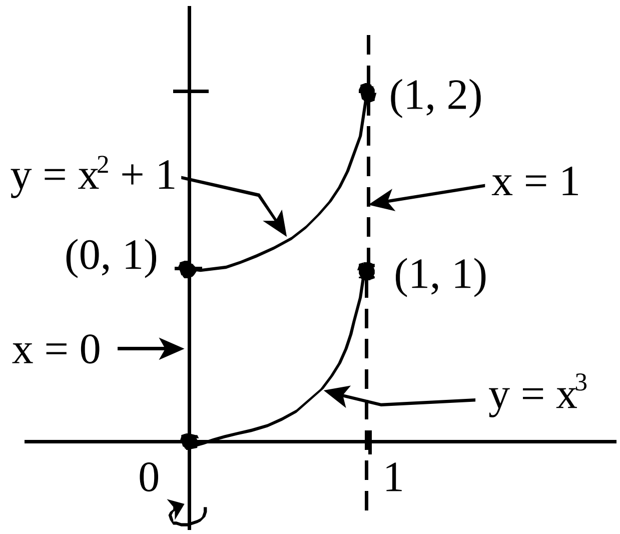
   3. Convert remaining sines into cosines using $\sin^2(x) = 1 - \cos^2(x)$

      $= \int \underbrace{\left(1-\cos^2(x)\right)}_{1-u^2} \underbrace{\cos^4(x)}_{u^4} \underbrace{\sin(x)\,dx}_{-du} = \int (1-u^2)\,u^4(-du) = \int (u^2-1)\,u^4 du$

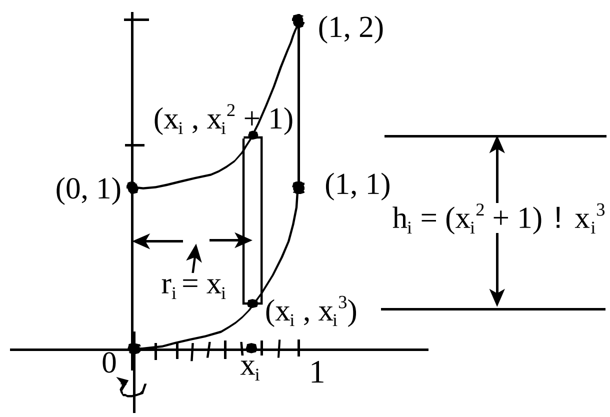
      $= \int (u^6 - u^4)\,du = \frac{1}{7}u^7 - \frac{1}{5}u^5 + C = \frac{1}{7}\cos^7(x) - \frac{1}{5}\cos^5(x) + C$

9. Use the Shell Method to compute the volume of the solid of revolution generated by revolving the region bounded by the graphs $y = x^3$, $y = x^2 + 1$, $x = 0$, and $x = 1$ about the y-axis.

   1. Sketch the bounded region



   2. Inscribe a rectangle parallel to the axis of revolution, and partition the interval spanned by the rectangles.



   3. Compute the vol of the i^{th} shell

      $vol_i \approx 2\pi r_i h_i \Delta x = 2\pi x_i \left(x_i^2 + 1 - x_i^3\right) \Delta x = 2\pi \left(x_i^3 + x_i - x_i^4\right) \Delta x$

   4. Approximate the volume of the solid of revolution by adding up the volumes of the shells

      $Vol \approx \sum_{i=1}^{n} 2\pi \left(x_i^3 + x_i - x_i^4\right) \Delta x$

   5. Let $\Delta x \to 0$

      $Vol = \lim_{\Delta x \to 0} \sum_{i=1}^{n} 2\pi \left(x_i^3 + x_i - x_i^4\right) \Delta x = \int_{x=0}^{x=1} 2\pi \left(x^3 + x - x^4\right) dx$

$$= 2\pi \left[\frac{1}{4}x^4 + \frac{1}{2}x^2 - \frac{1}{5}x^5 \right]_{x=0}^{x=1}$$

$$= 2\pi \left[\left(\frac{1}{4}\left(1\right)^4 + \frac{1}{2}\left(1\right)^2 - \frac{1}{5}\left(1\right)^5 \right) - \left(\frac{1}{4}\left(0\right)^4 + \frac{1}{2}\left(0\right)^2 - \frac{1}{5}\left(0\right)^5 \right) \right] = \frac{11\pi}{10}$$