

MTH 1125 Test #1 - (1 pm class) - Solutions
FALL 2018

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Name _____

Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x \rightarrow 3} \frac{x^2+6x-12}{x^2+2x+12} =$

Step #1 Try Plugging In:

$$\lim_{x \rightarrow 3} \frac{x^2+6x-12}{x^2+2x+12} = \frac{(3)^2+6(3)-12}{(3)^2+2(3)+12} = \frac{15}{27} = \frac{5}{9}$$

i.e., $\lim_{x \rightarrow 3} \frac{x^2+6x-12}{x^2+2x+12} = \frac{5}{9}$

2. Compute: $\lim_{x \rightarrow 6} \frac{x^2-9x+18}{x^2-5x-6} =$

Step #1 Try Plugging in:

$$\lim_{x \rightarrow 6} \frac{x^2-9x+18}{x^2-5x-6} = \frac{(6)^2-9(6)+18}{(6)^2-5(6)-6} = \frac{0}{0} \quad \begin{array}{l} \text{No Good -} \\ \text{Zero Divide!} \end{array}$$

Step #2 Try Factoring and Cancelling:

$$\lim_{x \rightarrow 6} \frac{x^2-9x+18}{x^2-5x-6} = \lim_{x \rightarrow 6} \frac{(x-3)(x-6)}{(x+1)(x-6)} = \lim_{x \rightarrow 6} \frac{(x-3)}{(x+1)} = \frac{(6)-3}{(6)+1} = \frac{3}{7}$$

i.e., $\lim_{x \rightarrow 6} \frac{x^2-9x+18}{x^2-5x-6} = \frac{3}{7}$

3. Compute: $\lim_{x \rightarrow 2} \frac{x^2+4x-9}{x^2+4x-12} =$

Step #1 Try Plugging in:

$$\lim_{x \rightarrow 2} \frac{x^2+4x-9}{x^2+4x-12} = \frac{(2)^2+4(2)-9}{(2)^2+4(2)-12} = \frac{3}{0} \quad \begin{array}{l} \text{No Good -} \\ \text{Zero Divide!} \end{array}$$

Step #2 Try Factoring and Cancelling:

No Good! “Factoring and Cancelling” only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x \rightarrow 2^-} \frac{x^2+4x-9}{x^2+4x-12} = \lim_{x \rightarrow 2^-} \frac{x^2+4x-9}{(x+6)(x-2)} = \frac{3}{(8)(-\varepsilon)} = \frac{\left(\frac{3}{8}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{l} x \rightarrow 2^- \\ \Rightarrow x < 2 \\ \Rightarrow x - 2 < 0 \end{array}$$

$$\lim_{x \rightarrow 2^+} \frac{x^2+4x-9}{x^2+4x-12} = \lim_{x \rightarrow 2^+} \frac{x^2+4x-9}{(x+6)(x-2)} = \frac{3}{(8)(+\varepsilon)} = \frac{\left(\frac{3}{8}\right)}{(+\varepsilon)} = +\infty$$

$$\begin{array}{l} x \rightarrow 2^+ \\ \Rightarrow x > 2 \\ \Rightarrow x - 2 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x \rightarrow 2} \frac{x^2+4x-9}{x^2+4x-12}$ **Does Not Exist!**

4. Compute: $\lim_{x \rightarrow -\infty} \frac{3x^6+4x^4-5}{x^4+4x^3-8x} =$

$$\lim_{x \rightarrow -\infty} \frac{3x^6+4x^4-5}{x^4+4x^3-8x} = \lim_{x \rightarrow -\infty} \frac{3x^6}{x^4} = \lim_{x \rightarrow -\infty} 3x^2 = +\infty$$

$$\text{i.e., } \lim_{x \rightarrow -\infty} \frac{3x^6+4x^4-5}{x^4+4x^3-8x} = +\infty$$

5. $f(x) = \frac{x^2+4x+3}{x^2-3x-10}$ Find the asymptotes and graph

Verticals

1. Find x -values that cause division by zero.

$$\Rightarrow x^2 - 3x - 10 = 0$$

$$\Rightarrow (x + 2)(x - 5) = 0$$

$\Rightarrow x = -2$ and $x = 5$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$$\lim_{x \rightarrow -2^-} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x \rightarrow -2^-} \frac{x^2+4x+3}{(x+2)(x-5)} = \frac{-1}{(-\varepsilon)(-7)} = \frac{1}{(-\varepsilon)(7)} = \frac{(\frac{1}{7})}{-\varepsilon} = -\infty$$

$$\begin{array}{l} x \rightarrow -2^- \\ \Rightarrow x < -2 \\ \Rightarrow x + 2 < 0 \end{array}$$

$$\lim_{x \rightarrow -2^+} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x \rightarrow -2^+} \frac{x^2+4x+3}{(x+2)(x-5)} = \frac{-1}{(\varepsilon)(-7)} = \frac{1}{(\varepsilon)(7)} = \frac{(\frac{1}{7})}{\varepsilon} = +\infty$$

$$\begin{array}{l} x \rightarrow -2^+ \\ \Rightarrow x > -2 \\ \Rightarrow x + 2 > 0 \end{array}$$

Since the one-sided limits are infinite, $x = -2$ is a vertical asymptote.

$$\lim_{x \rightarrow 5^-} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x \rightarrow 5^-} \frac{x^2+4x+3}{(x+2)(x-5)} = \frac{48}{(7)(-\varepsilon)} = \frac{(\frac{48}{7})}{(-\varepsilon)} = -\infty$$

$$\begin{array}{l} x \rightarrow 5^- \\ \Rightarrow x < 5 \\ \Rightarrow x - 5 < 0 \end{array}$$

$$\lim_{x \rightarrow 5^+} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x \rightarrow 5^+} \frac{x^2+4x+3}{(x+2)(x-5)} = \frac{48}{(7)(\varepsilon)} = \frac{(\frac{48}{7})}{(\varepsilon)} = +\infty$$

$$\begin{array}{l} x \rightarrow 5^+ \\ \Rightarrow x > 5 \\ \Rightarrow x - 5 > 0 \end{array}$$

Since the one-sided limits are **infinite**, $x = 5$ is a vertical asymptote.

Horizontals

Compute the limits as $x \rightarrow -\infty$ and as $x \rightarrow +\infty$

$$\lim_{x \rightarrow -\infty} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x \rightarrow -\infty} \frac{x^2}{x^2} = \lim_{x \rightarrow -\infty} 1 = 1$$

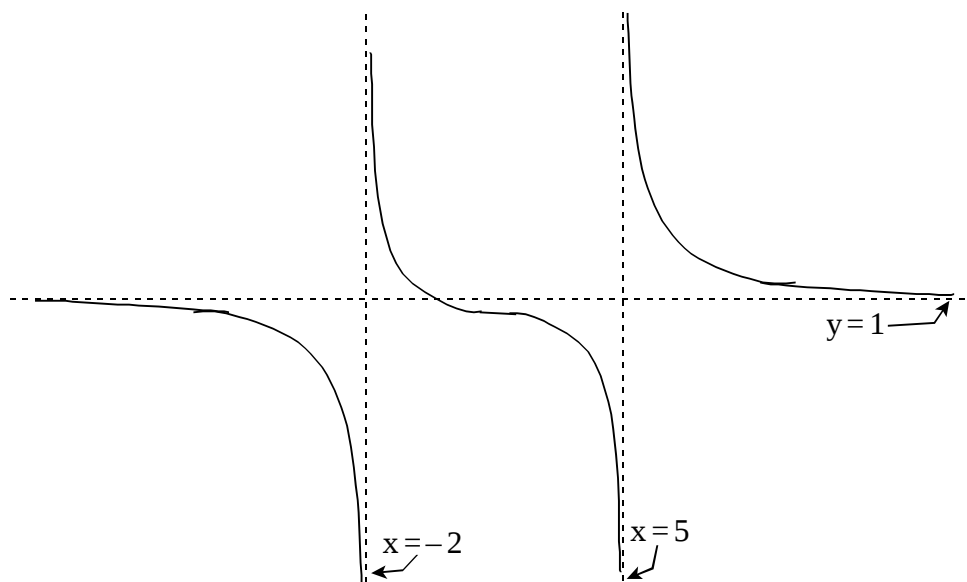
$$\lim_{x \rightarrow +\infty} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x \rightarrow +\infty} \frac{x^2}{x^2} = \lim_{x \rightarrow +\infty} 1 = 1$$

Since the limits are **finite** and **constant**, $y = 1$ is a horizontal asymptote.

Summary:

$\lim_{x \rightarrow -2^-} \frac{x^2+4x+3}{x^2-3x-10} = -\infty$	$\lim_{x \rightarrow -\infty} \frac{x^2+4x+3}{x^2-3x-10} = 1$ $\lim_{x \rightarrow +\infty} \frac{x^2+4x+3}{x^2-3x-10} = 1$
$\lim_{x \rightarrow -2^+} \frac{x^2+4x+3}{x^2-3x-10} = +\infty$	
$\lim_{x \rightarrow 5^-} \frac{x^2+4x+3}{x^2-3x-10} = -\infty$	
$\lim_{x \rightarrow 5^+} \frac{x^2+4x+3}{x^2-3x-10} = +\infty$	

Graph $f(x) = \frac{x^2+4x+3}{x^2-3x-10}$



6. Compute: $\lim_{x \rightarrow 1} \frac{\sqrt{x+8}-3}{x-1} =$

Step #1 Try Plugging in:

$$\lim_{x \rightarrow 1} \frac{\sqrt{x+8}-3}{x-1} = \frac{\sqrt{(1)+8}-3}{(1)-1} = \frac{0}{0} \quad \begin{array}{l} \text{No Good -} \\ \text{Zero Divide!} \end{array}$$

Step #2 Try Factoring and Cancelling:

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{\sqrt{x+8}-3}{x-1} &= \lim_{x \rightarrow 1} \frac{\sqrt{x+8}-3}{x-1} \cdot \frac{\sqrt{x+8}+3}{\sqrt{x+8}+3} = \lim_{x \rightarrow 1} \frac{(\sqrt{x+8})^2 - (3)^2}{(x-1)(\sqrt{x+8}+3)} \\ &= \lim_{x \rightarrow 1} \frac{(x+8)-9}{(x-1)(\sqrt{x+8}+3)} = \lim_{x \rightarrow 1} \frac{(x-1)}{(x-1)(\sqrt{x+8}+3)} = \lim_{x \rightarrow 1} \frac{1}{\sqrt{x+8}+3} \\ &= \frac{1}{\sqrt{(1)+8}+3} = \frac{1}{3+3} = \frac{1}{6} \end{aligned}$$

i.e., $\lim_{x \rightarrow 1} \frac{\sqrt{x+8}-3}{x-1} = \frac{1}{6}$

7.

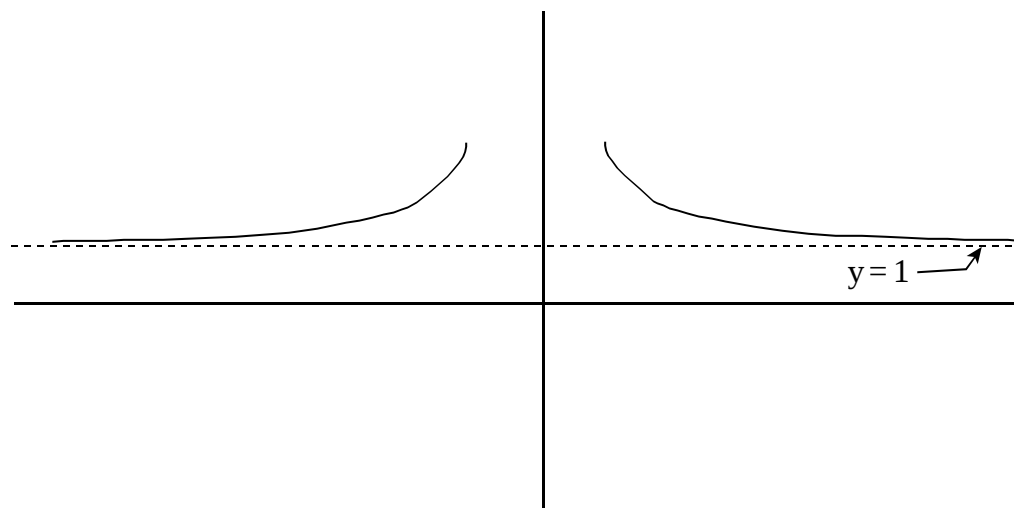
$x =$	$f(x) =$	$x =$	$f(x) =$
-10	10.87	10	10.87
-100	1.12	100	1.12
-1000	1.009	1000	1.009
-10,000	1.0023	10,000	1.0023
-100,000	1.00009	100,000	1.00009

Based on the information in the table above, do the following:

(a) $\lim_{x \rightarrow -\infty} f(x) = 1$

(b) $\lim_{x \rightarrow +\infty} f(x) = 1$

(c) Graph $f(x)$



Extra: (Wow! 10 points) Show CLEARLY how you arrive at your answer!

Compute: $\lim_{x \rightarrow -\infty} \frac{3x+5}{\sqrt{x^2+6x+9}}$

$$\lim_{x \rightarrow -\infty} \frac{3x+5}{\sqrt{x^2+6x+9}}$$

$$= \lim_{x \rightarrow -\infty} \frac{3x}{\sqrt{x^2}}$$

$$\swarrow \quad \left(\sqrt{x^2} = |x| \right)$$

$$= \lim_{x \rightarrow -\infty} \frac{3x}{|x|}$$

$$\swarrow$$

$$\swarrow$$

(Since $x \rightarrow -\infty$, we can assume that $x < 0$. Hence, $|x| = -x$)

$$= \lim_{x \rightarrow -\infty} \frac{3x}{-x}$$

$$\swarrow$$

$$= \lim_{x \rightarrow -\infty} (-3) = -3$$

$$\lim_{x \rightarrow -\infty} \frac{3x+5}{\sqrt{x^2+6x+9}} = -3$$