MTH 3318 - Test #2 - Solutions

Spring 2013

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Instructions. Fully document your work.

1. In exercises 1.a - 1.d, let p be the statement: "He pays me," and let q be the statement: "I will do the work." Write each statement in symbolic form.

   (a) If $\underbrace{\text{he pays me,}}_{p}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{\text{I will do the work.}}_{q}$

   $p \rightarrow q$

   (b) $\underbrace{\text{He will pay me,}}_{p}\ \underbrace{\text{or}}_{\vee}\ \underbrace{\text{I will not do the work.}}_{\sim q}$

   $p \vee \sim q$

   (c) $\underbrace{\text{His paying me}}_{p}\ \underbrace{\text{is a necessary and sufficient condition for}}_{\leftrightarrow}\ \underbrace{\text{me to do the work.}}_{q}$

   $p \leftrightarrow q$

   (d) He will pay me if I do the work. $\underbrace{\text{He will pay me}}_{p}\ \underbrace{\text{if}}_{\leftarrow}\ \underbrace{\text{I do the work.}}_{q}$

   $p \leftarrow q$ or $q \rightarrow p$

2. In exercises 2.a - 2.d, let p be the statement: "I will buy new clothes," and let q be the statement: "I will look good." Write each statement in words.

   (a) $p \wedge q$

   $\underbrace{\text{I will buy new clothes}}_{p}\ \underbrace{\text{and}}_{\wedge}\ \underbrace{\text{I will look good.}}_{q}$

   (b) $p \vee q$

   $\underbrace{\text{I will buy new clothes}}_{p}\ \underbrace{\text{or}}_{\vee}\ \underbrace{\text{I will look good.}}_{q}$

   (c) $q \rightarrow \sim p$

   If $\underbrace{\text{I look good,}}_{q}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{\text{I will not buy new clothes.}}_{\sim p}$

(d) $\sim p \quad \leftrightarrow \sim q$

$\underbrace{\text{I will not buy new clothes}}_{\sim p}\ \underbrace{\text{if and only if}}_{\leftrightarrow}\ \underbrace{\text{I do not look good.}}_{\sim q}$

3. In problems 3.a - 3.d, determine whether the given propositions are True or False:

(a) $\underbrace{\text{If } 8+3=9,}_{F}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{8>10.}_{F}$

$F \rightarrow F \equiv T$

The proposition is TRUE.

(b) If $\underbrace{8>3,}_{T}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{8>5.}_{T}$

$T \rightarrow T = T$

The proposition is TRUE.

(c) If $\underbrace{8>10,}_{F}\ \underbrace{\text{if and only if}}_{\leftrightarrow}\ \underbrace{2+2=5.}_{F}$

$F \leftrightarrow F = T$

The proposition is TRUE.

(d) If $\underbrace{2+2=5,}_{F}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{8>10.}_{F}$

$F \rightarrow F = T$

The proposition is TRUE.

4. In exercises 4.a-4.b construct a truth table for the statement given.

(a) $p \wedge (q \longleftrightarrow r)$

p	q	r	$(q \longleftrightarrow r)$	$p \wedge (q \longleftrightarrow r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	T	T
F	T	T	T	F
F	T	F	F	F
F	F	T	F	F
F	F	F	T	F

(b) $(\sim p \wedge q) \rightarrow \sim r$

p	q	r	$\sim p$	$\sim r$	$(\sim p \wedge q)$	$(\sim p \wedge q) \rightarrow \sim r$
T	T	T	F	F	F	T
T	T	F	F	T	F	T
T	F	T	F	F	F	T
T	F	F	F	T	F	T
F	T	T	T	F	T	F
F	T	F	T	T	T	T
F	F	T	T	F	F	T
F	F	F	T	T	F	T

5. For problems 5.a - 5.d, negate the given statements:

(a) All kids have freckles.

Possible negations:

Some kids don't have freckles.

At least one kid doesn't have freckles.

There exists a kid that doesn't have freckles.

(b) No men snore.

Possible negations:

Some men snore.

At least one man snores.

There exists a man that snores.

(c) Some flowers have nectar.

Possible negations:

No flowers have nectar.

There does not exist a flower that has nectar.

(d) $\forall$ real numbers x, $\exists$ real number y, $\ni y = \frac{1}{x}$.
(i.e. For all real numbers x, there exists a number y such that $y = \frac{1}{x}$.)

$\sim \left(\forall \text{ real numbers } x, \exists \text{ a real number } y, \ni\ y = \frac{1}{x}.\right)$

$\equiv \exists \text{ a real number } x, \ni \sim \left(\exists \text{ a real number } y, \ni\ y = \frac{1}{x}.\right)$

$\equiv \exists \text{ a real number } x, \ni \forall \text{ real numbers } y, \sim \left(y = \frac{1}{x}.\right)$

$\equiv \exists \text{ a real number } x, \ni \forall \text{ real numbers } y,\ y \neq \frac{1}{x}.$

$\exists$ a real number x, $\ni$ $\forall$ real numbers y, $y \neq \frac{1}{x}$.

6. For problems 6.a - 6.b, disprove the given statements by providing a suitable counter-example:

   (a) $\forall n \in \mathbb{N}$, if $2n$ is even, then n is also even.

   Counter-example:

   Let $n = 3$.

   Then $2n = 6$ is even, but n is odd.

   Hence our statement if false by counter-example.

   (b) For all integers x, y, and z, if x is a factor of $(y + z)$, then x is a factor of y and x is a factor of z.

   Counter-example:

   Let $x = 2$, $y = 3$, and $z = 5$.

   Then x is a factor of $(y + z)$, but x is neither a factor of y or z.

7. Write the converse, inverse, and contrapositive of the following statement, labeling each one.

   If I turn the key, then the car will start.

   $$\text{If } \underbrace{\text{I turn the key,}}_{p} \underbrace{\text{then}}_{\rightarrow} \underbrace{\text{the car will start.}}_{q}$$

   converse:

   $$\text{If } \underbrace{\text{the car starts,}}_{q} \underbrace{\text{then}}_{\rightarrow} \underbrace{\text{I will turn the key.}}_{p}$$

   inverse:

   $$\text{If } \underbrace{\text{I do not turn the key,}}_{\sim p} \underbrace{\text{then}}_{\rightarrow} \underbrace{\text{the car will not start.}}_{\sim q}$$

   contrapositive:

   $$\text{If } \underbrace{\text{the car will not start,}}_{\sim q} \underbrace{\text{then}}_{\rightarrow} \underbrace{\text{I will not turn the key.}}_{\sim p}$$

8. In problems 8.a - 8.b, determine whether the given arguments are valid.

(a) I will make him a partner if and only if he closes this deal. If he leaves tonight, then he will close this deal. Therefore, I will make him a partner if he leaves tonight.

If we make the following assignments:

p : I will make him a partner.

r : He will close this deal.

s : He will leave tonight.

Then our argument has the form:

p_1 : $\underbrace{\text{I will make him a partner}}_{p}\ \underbrace{\text{if and only if}}_{\leftrightarrow}\ \underbrace{\text{he closes this deal.}}_{r}$

p_2 : If $\underbrace{\text{he leaves tonight,}}_{s}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{\text{he will close this deal.}}_{r}$

q : $\underbrace{\text{Therefore,}}_{\therefore}\ \underbrace{\text{I will make him a partner}}_{p}\ \underbrace{\text{if}}_{\leftarrow}\ \underbrace{\text{he leaves tonight.}}_{s}$

i.e., Our argument has the form: $(p_1 \wedge p_2) \rightarrow q$

p	r	s	$p_1 : (p \leftrightarrow r)$	$p_2 : (s \rightarrow r)$	$(p_1 \wedge p_2)$	$q : (s \rightarrow p)$	$(p_1 \wedge p_2) \rightarrow q$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	T
T	F	F	F	T	F	T	T
F	T	T	F	T	F	F	T
F	T	F	F	T	F	T	T
F	F	T	T	F	F	F	T
F	F	F	T	T	T	T	T

Note the argument is a tautology. Therefore, it is VALID.

(b) Some dolphins are fish. All fish taste good. Therefore, some dolphins taste good.

If we make the following assignments:

$D-$ Dolphins

$F-$ Fish

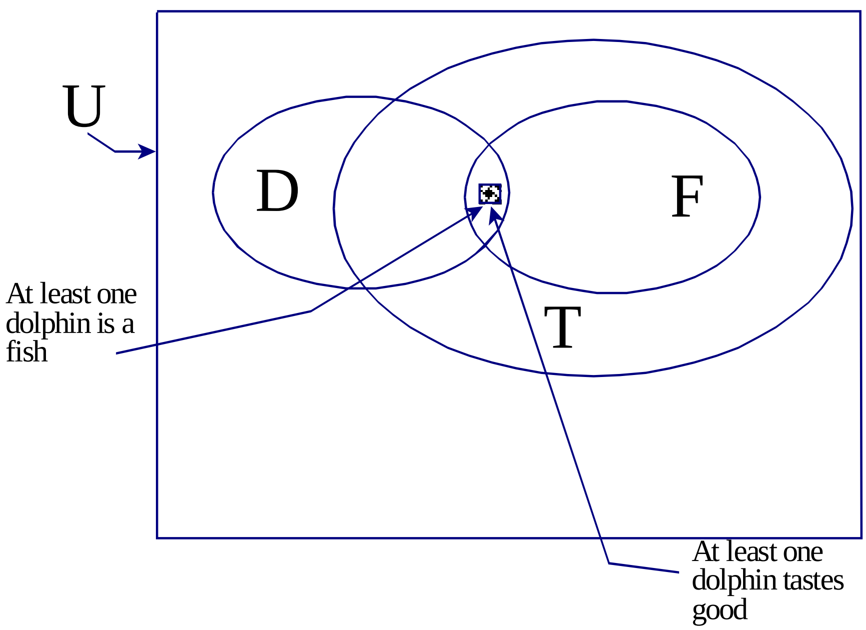
$T-$ Things that Taste Good

Then our argument has the form:

p_1 : Some dolphins are fish.

p_2 : All fish taste good.

$\therefore q$: Some dolphins taste good.



Note that if the premises are true, it is impossible for the conclusion to be false.

Therefore, the argument is VALID.

9. In problems 9.a - 9.b, determine whether the given arguments are valid.

   (a) If I eat right and I exercise, then I will make the team. I will make the team. Therefore, if I don't make the team, then I don't eat right.

   If we make the following assignments:

   p : I will eat right.

   r : I will exercise.

   s : I will make the team.

   Then our argument has the form:

   $$p_1 : \text{If } \underbrace{\text{I eat right}}_{(p} \underbrace{\text{and}}_{\wedge} \underbrace{\text{I exercise,}}_{r)} \underbrace{\text{then}}_{\rightarrow} \underbrace{\text{I will make the team.}}_{s}$$

   $$p_2 : \underbrace{\text{I will make the team.}}_{s}$$

   $$q : \underbrace{\text{Therefore,}}_{\therefore} \text{ if } \underbrace{\text{I don't make the team,}}_{\sim s} \underbrace{\text{then}}_{\rightarrow} \underbrace{\text{I don't eat right.}}_{\sim p}$$

   i.e., Our argument has the form: $(p_1 \wedge p_2) \rightarrow q$

p	r	s	$\sim p$	$\sim s$	$p \wedge r$	$p_1 : (p \wedge r) \rightarrow s$	$p_2 : s$	$(p_1 \wedge p_2)$	$q : \sim s \rightarrow \sim p$	$(p_1 \wedge p_2) \rightarrow q$
T	T	T	F	F	T	T	T	T	T	T
T	T	F	F	T	T	F	F	F	F	T
T	F	T	F	F	F	T	T	T	T	T
T	F	F	F	T	F	T	F	F	F	T
F	T	T	T	F	F	T	T	T	T	T
F	T	F	T	T	F	T	F	F	T	T
F	F	T	T	F	F	T	T	T	T	T
F	F	F	T	T	F	T	F	F	T	T

   Note the argument is a tautology. Therefore, it is VALID.

(b) No squares are rectangles. Some triangles are rectangles. Therefore, no squares are rectangles.

If we make the following assignments:

$S-$ Squares

$R-$ Rectangles

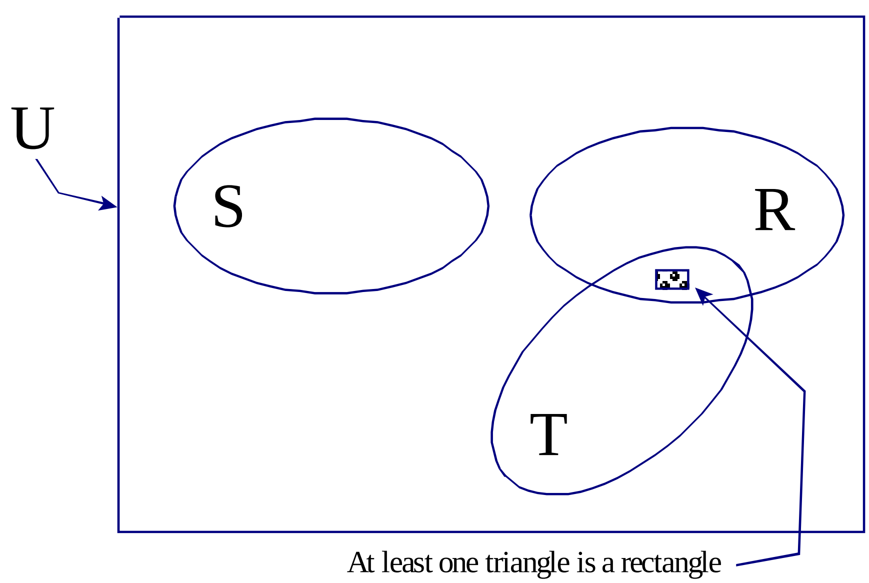
$T-$ Triangles

Then our argument has the form:

p_1 : No squares are rectangles.

p_2 : Some triangles are rectangles.

$\therefore q$: No squares are rectangles.



Note that if the premises are true, it is impossible for the conclusion to be false (Since the conclusion IS one of the premises).

Therefore, the argument is VALID.