MTH 1125 Test #1 - (11 am class) - Solutions

FALL 2019

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 2} \frac{2x^2+x+4}{x^2+5x-12} =$

Step #1 Try Plugging In:

$$\lim_{x\to 2} \frac{2x^2+x+4}{x^2+5x-12} = \frac{2(2)^2+(2)+4}{(2)^2+5(2)-12} = \frac{14}{2} = 7$$

i.e., $\lim_{x\to 2} \frac{2x^2+x+4}{x^2+5x-12} = 7$

2. Compute: $\lim_{x\to 6} \frac{x^2-5x-6}{x^2-9x+18} =$

$$\lim_{x\to 6} \frac{x^2-5x-6}{x^2-9x+18} = \frac{(6)^2-5(6)-6}{(6)^2-9(6)+18} = \frac{0}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 6} \frac{x^2-5x-6}{x^2-9x+18} = \lim_{x\to 6} \frac{(x+1)(x-6)}{(x-3)(x-6)} = \lim_{x\to 6} \frac{(x+1)}{(x-3)} = \frac{(6)+1}{(6)-3} = \frac{7}{3}$$

i.e., $\lim_{x\to 6} \frac{x^2-5x-6}{x^2-9x+18} = \frac{7}{3}$

3. Compute: $\lim_{x\to\ 3} \frac{x^2+4x-9}{x^2-2x-3} =$

Step #1 Try Plugging in:

$$\lim_{x\to\ 3} \frac{x^2+4x-9}{x^2-2x-3} = \frac{(3)^2+4(3)-9}{(3)^2-2(3)-3} = \frac{12}{0}$$

No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

No Good! "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to\ 3^-} \frac{x^2+4x-9}{x^2-2x-3} = \lim_{x\to\ 3^-} \frac{x^2+4x-9}{(x+1)(x-3)} = \frac{12}{(4)(-\varepsilon)} = \frac{3}{(-\varepsilon)} = -\infty$$

$$\begin{aligned} & x \to 3^- \\ \Rightarrow\quad & x < 3 \\ \Rightarrow\quad & x - 3 < 0 \end{aligned}$$

$$\lim_{x\to\ 3^+} \frac{x^2+4x-9}{x^2-2x-3} = \lim_{x\to\ 3^+} \frac{x^2+4x-9}{(x+1)(x-3)} = \frac{12}{(4)(+\varepsilon)} = \frac{3}{(+\varepsilon)} = +\infty$$

$$\begin{aligned} & x \to 3^+ \\ \Rightarrow\quad & x > 3 \\ \Rightarrow\quad & x - 3 > 0 \end{aligned}$$

Since the one-sided limits are not equal, $\lim_{x\to\ 3} \frac{x^2+4x-9}{x^2-2x-3}$ **Does Not Exist!**

4. Compute: $\lim_{x\to\ -\infty} \frac{3x^6+7x^2-5}{x^4+6x^3-8x} =$

$$\lim_{x\to\ -\infty} \frac{3x^6+7x^2-5}{x^4+6x^3-8x} = \lim_{x\to\ -\infty} \frac{3x^6}{x^4} = \lim_{x\to\ -\infty} 3x^2 = +\infty$$

i.e., $\lim_{x\to\ -\infty} \frac{3x^6+7x^2-5}{x^4+6x^3-8x} = +\infty$

5. $f(x) = \frac{x^2-9}{x^2+x-20}$ Find the asymptotes and graph

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow x^2 + x - 20 = 0$

$\Rightarrow (x+5)(x-4) = 0$

$\Rightarrow x = -5$ and $x = 4$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$$\lim_{x\to\ -5^-} \frac{x^2-9}{x^2+x-20} = \lim_{x\to\ -5^-} \frac{x^2-9}{(x+5)(x-4)} = \frac{16}{(-\varepsilon)(-9)} = \frac{16}{(\varepsilon)(9)} = \frac{\left(\frac{16}{9}\right)}{\varepsilon} = +\infty$$

$$\begin{array}{|ll|} & x \to -5^- \\ \Rightarrow & x < -5 \\ \Rightarrow & x+5 < 0 \end{array}$$

$$\lim_{x\to\ -5^+} \frac{x^2-9}{x^2+x-20} = \lim_{x\to\ -5^+} \frac{x^2-9}{(x+5)(x-4)} = \frac{16}{(+\varepsilon)(-9)} = \frac{\left(-\frac{16}{9}\right)}{\varepsilon} = -\infty$$

$$\begin{array}{|ll|} & x \to -5^+ \\ \Rightarrow & x > -5 \\ \Rightarrow & x+5 > 0 \end{array}$$

Since the one-sided limits are infinite, $x = -5$ *is a vertical asymptote.*

$$\lim_{x\to\ 4^-} \frac{x^2-9}{x^2+x-20} = \lim_{x\to\ 4^-} \frac{x^2-9}{(x+5)(x-4)} = \frac{7}{(9)(-\varepsilon)} = \frac{\left(\frac{7}{9}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{|ll|} & x \to 4^- \\ \Rightarrow & x < 4 \\ \Rightarrow & x-4 < 0 \end{array}$$

$$\lim_{x\to 4^+} \frac{x^2-9}{x^2+x-20} = \lim_{x\to 4^+} \frac{x^2-9}{(x+5)(x-4)} = \frac{7}{(9)(+\varepsilon)} = \frac{\left(\frac{7}{9}\right)}{(\varepsilon)} = +\infty$$

$$\begin{array}{|ll|} & x \to 4^+ \\ \Rightarrow & x > 4 \\ \Rightarrow & x-4 > 0 \end{array}$$

Since the one-sided limits are **infinite,** $x = 4$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{x^2-9}{x^2+x-20} = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1$

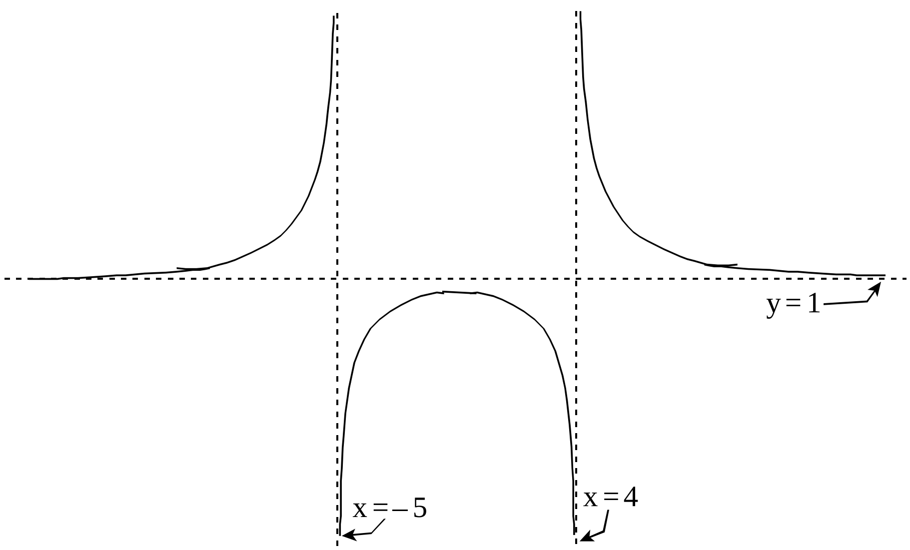
$\lim_{x\to+\infty} \frac{x^2-9}{x^2+x-20} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are **finite** and **constant,** $y = 1$ *is a horizontal asymptote.*

Summary:

$\lim_{x\to-5^-} \frac{x^2-9}{x^2+x-20} = +\infty$	
$\lim_{x\to-5^+} \frac{x^2-9}{x^2+x-20} = -\infty$	$\lim_{x\to-\infty} \frac{x^2-9}{x^2+x-20} = 1$
$\lim_{x\to4^-} \frac{x^2-9}{x^2+x-20} = -\infty$	$\lim_{x\to+\infty} \frac{x^2-9}{x^2+x-20} = 1$
$\lim_{x\to4^+} \frac{x^2-9}{x^2+x-20} = +\infty$	

Graph $f(x) = \frac{x^2-9}{x^2+x-20}$



6. Compute: $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} =$

Step #1 Try Plugging in:

$$\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} = \frac{\sqrt{(4)+5}-3}{(4)-4} = \frac{0}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} = \lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} \cdot \frac{\sqrt{x+5}+3}{\sqrt{x+5}+3} = \lim_{x\to 4} \frac{\left(\sqrt{x+5}\right)^2-(3)^2}{(x-4)\left[\sqrt{x+5}+3\right]}$$

$$= \lim_{x\to 4} \frac{(x+5)-9}{(x-4)\left[\sqrt{x+5}+3\right]} = \lim_{x\to 4} \frac{(x-4)}{(x-4)\left[\sqrt{x+5}+3\right]} = \lim_{x\to 4} \frac{1}{\left[\sqrt{x+5}+3\right]}$$

$$= \frac{1}{\left[\sqrt{(4)+5}+3\right]} = \frac{1}{[3+3]} = \frac{1}{6}$$

$$\boxed{\text{i.e., } \lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} = \frac{1}{6}}$$

7.

$x =$	$f(x) =$
-2.5	3.6
-2.1	30.8
-2.01	318.9
-2.001	$3,241.9$
-2.0001	$35,342.2$

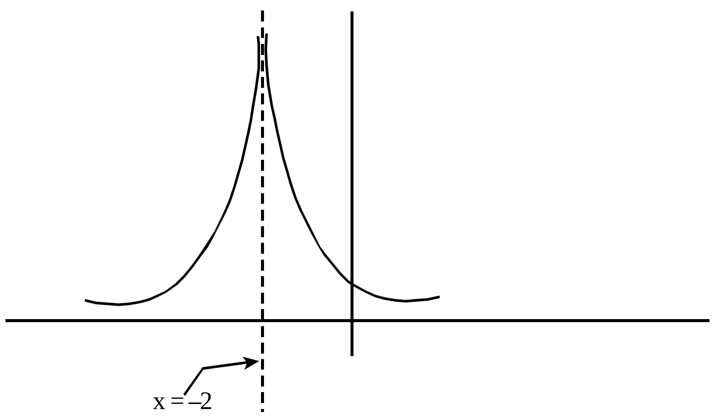
$x =$	$f(x) =$
-1.5	3.6
-1.9	30.8
-1.99	318.9
-1.999	$3,241.9$
-1.9999	$35,342.2$

Based on the information in the table above, do the following:

(a) $\lim_{x \to -2^-} f(x) = +\infty$

(b) $\lim_{x \to -2^+} f(x) = +\infty$

(c) Graph $f(x)$



8. Determine whether or not $f(x)$ is continuous at the point $x = 2$.

$$f(x) = \begin{cases} 3x - 2 & \text{for } x < 2 \\ 6 & \text{for } x = 2 \\ 5x - 4 & \text{for } x > 2 \end{cases}$$

If $f(x)$ is continuous at the point $x = 2$, then $\lim_{x \to 2} f(x) = f(2)$.

To see if this is true, we'll compute $\lim_{x \to 2} f(x)$.

Since the definition of $f(x)$ changes at $x = 2$, we must compute the one-sided limits in order to determine whether the limit exists.

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (3x - 2) = 3(2) - 2 = 4$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (5x - 4) = 5(2) - 4 = 6$

Since the one-sided limits are NOT equal, $\lim_{x \to 2} f(x)$ Does Not Exist.

Hence: $\lim_{x \to 2} f(x) \neq f(2)$

Therefore, $f(x)$ is NOT continuous at $x = 2$