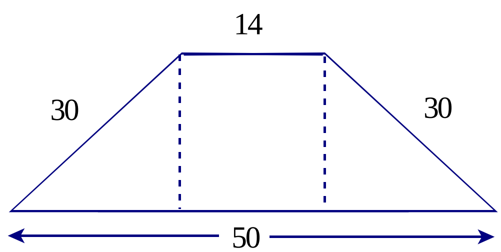
P. 80 - Exercises and Solutions

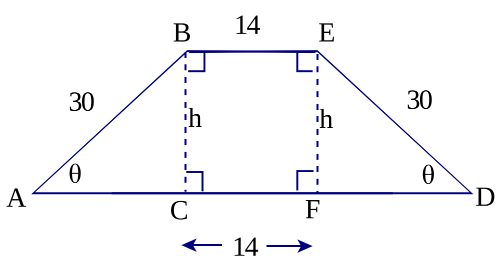
TERM 5 2022

Pat Rossi Name ______________________________

Ex 2 - An Old Babylonian tablet calls for finding the area of an isosceles trapezoid whose sides are 30 units long and whose bases are 14 and 50 units. Solve this problem. (Note: an isosceles trapezoid is a trapezoid whose non-parallel sides are equal and whose base angles are equal.)

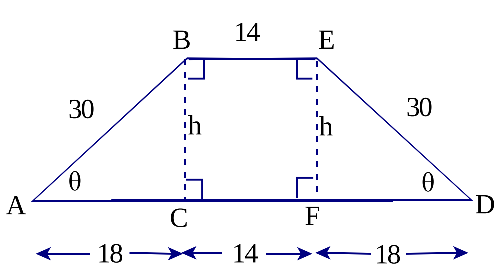


The top and base of the trapezoid are parallel. Hence the vertical sides of the triangles are equal, since they represent the distance from the base to the top. We'll call this distance h. The base angles of the trapezoid are equal by definition of "isosceles trapezoid." We'll denote these by θ. (See below.)

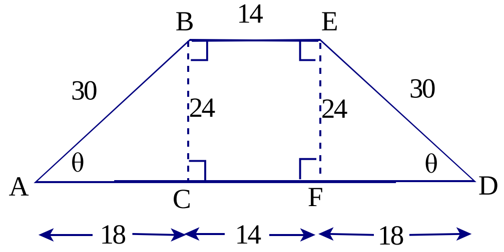


The remaining angle in each triangle is equal to $180^\circ - (90^\circ + \theta)$. ($180^\circ$ minus the measures of the other two angles.) Hence, the triangles are similar. As a consequence, the measures of corresponding sides of the triangle are proportional. Since the vertical side of each triangle is h, the proportion is 1. (i.e., the triangles are congruent.)

As another consequence, the length of line segment $\overline{AC}$ is equal to the length of line segment $\overline{DF}$. Hence, each has the length $\frac{50-14}{2} = 18$. (See below.)



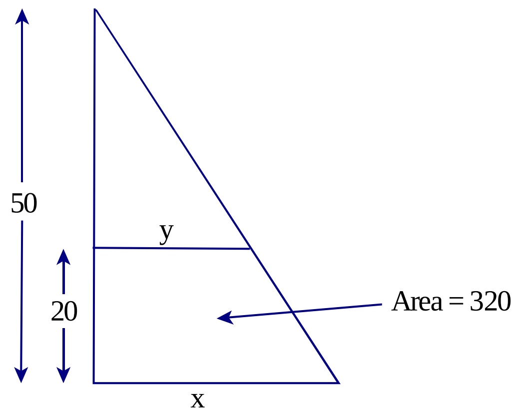
By Pythagorean's Theorem, $h^2 + 18^2 = 30^2, \Rightarrow h^2 = 30^2 - 18^2 \Rightarrow h^2 = 576 \Rightarrow h = 24$



The area of the trapezoid is equal to the sum of the areas of the two triangles plus the area of the central rectangle.

$A = \frac{1}{2}(18)(24) + \frac{1}{2}(18)(24) + (14)(24) = 768 \text{ units}^2$

Ex 2 - In another tablet, one side of a right triangle is 50 units long. Parallel to the other side, and 20 units from this side, a line is drawn that cuts off a right trapezoid of area 320 units^2. Find the lengths of the bases (i.e., the parallel sides) of the trapezoid. (Hint: If A is the area of the "original triangle" (i.e. the large triangle), then $320 + 15y = A = 25x$, and $\frac{1}{2}(x+y)\,20 = 320$.)



First, observe that the area of the "bottom triangle" (i.e., the "large triangle") is given by:

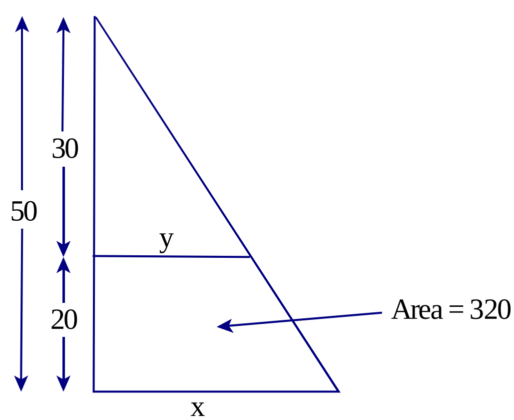
$A_{\text{large}} = \frac{1}{2}bh = \frac{1}{2}x \cdot 50 = 25x$

i.e., the area of the "large triangle" is $A_{\text{large}} = 25x$ (Eq. 1)

Next, observe that the area of the trapezoid is given by the average of the lengths of the parallel sides times the height

i.e. The area of the trapezoid is given by $A_{\text{trap}} = \frac{1}{2}(x+y) \cdot 20 = 320$

i.e. The area of the trapezoid is given by $A_{\text{trap}} = 10x + 10y = 320$ (Eq. 2)



The area of the "top triangle" is given by $A_{\text{top}} = \frac{1}{2}bh = \frac{1}{2}y \cdot 30 = 15y$

i.e., the area of the "top triangle" is $A_{\text{top}} = 15y$ (Eq. 3)

Finally, observe that the area of the "large triangle" is equal to the sum of the areas of the trapezoid and the "top triangle" i.e., the area of the "large triangle" is given by $A_{\text{large}} = \underbrace{320}_{A_{\text{trap}}} + \underbrace{15y}_{A_{\text{top}}}$ (Eq. 4)

From Eq. 1 and Eq. 4, we have: $A_{\text{large}} = 25x = 320 + 15y$

i.e., $25x = 320 + 15y$

$\Rightarrow 25x - 15y = 320$ (Eq. 5)

Eq. 2 and Eq, 5 give us a system of equation that we can solve of x and y

$10x + 10y = 320$

$25x - 15y = 320$

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$\Rightarrow 30x + 30y = 960$

$\Rightarrow 50x - 30y = 640$

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$\Rightarrow 80x \quad = 1600$

$\Rightarrow x = 20$
$\Rightarrow y = 12$