# MTH 1125 - Test 2 (12pm Class)

FALL 2021

Pat Rossi                                  Name ______________________________

**Instructions.** Show CLEARLY how you arrive at your answers.

1. Compute: $\frac{d}{dx}\left[4x^5+4x^4+6x^3+6x^2+8x+8\sqrt{x}+10\right]=$

$\frac{d}{dx}\left[4x^5+4x^4+6x^3+6x^2+8x+8\sqrt{x}+10\right]$

$=4\left[5x^4\right]+4\left[4x^3\right]+6\left[3x^2\right]+6\left[2x\right]+8+8\left[\frac{1}{2}x^{-\frac{1}{2}}\right]+0$

$=20x^4+16x^3+18x^2+12x+8+4x^{-\frac{1}{2}}$

i.e., $\frac{d}{dx}\left[4x^5+4x^4+6x^3+6x^2+8x+8\sqrt{x}+10\right]=20x^4+16x^3+18x^2+12x+8+4x^{-\frac{1}{2}}$

2. Compute: $\frac{d}{dx}\left[\left(2x^3+\sec(x)\right)\left(5x^2+3x\right)\right]=$

$$\frac{d}{dx}\left[\underbrace{\left(2x^3+\sec(x)\right)}_{1^{st}}\underbrace{\left(5x^2+3x\right)}_{2^{nd}}\right]=\underbrace{\left(6x^2+\sec(x)\tan(x)\right)}_{1^{\text{st}}\text{ prime}}\cdot\underbrace{\left(5x^2+3x\right)}_{2^{nd}}+\underbrace{\left(10x+3\right)}_{2^{\text{nd}}\text{ prime}}\cdot\underbrace{\left(2x^3+\sec(x)\right)}_{1^{st}}$$

$\frac{d}{dx}\left[\left(2x^3+\sec(x)\right)\left(5x^2+3x\right)\right]=\left(6x^2+\sec(x)\tan(x)\right)\left(5x^2+3x\right)+\left(10x+3\right)\left(2x^3+\sec(x)\right)$

3. Compute: $\frac{d}{dx}\left[\frac{3x^5+6x^3+9x}{4x^2+1}\right]=$

$$\frac{d}{dx}\left[\frac{\overbrace{3x^5+6x^3+9x}^{\text{top}}}{\underbrace{4x^2+1}_{\text{Bottom}}}\right]=\frac{\overbrace{\left(15x^4+18x^2+9\right)}^{\text{top prime}}\cdot\overbrace{\left(4x^2+1\right)}^{\text{bottom}}-\overbrace{8x}^{\text{bottom prime}}\cdot\overbrace{\left(3x^5+6x^3+9x\right)}^{\text{top}}}{\underbrace{\left(4x^2+1\right)^2}_{\text{bottom squared}}}$$

i.e., $\frac{d}{dx}\left[\frac{3x^5+6x^3+9x}{4x^2+1}\right]=\frac{\left(15x^4+18x^2+9\right)\left(4x^2+1\right)-8x\left(3x^5+6x^3+9x\right)}{\left(4x^2+1\right)^2}$

4. Compute: $\frac{d}{dx}\left[\left(8x^3+12x^2+6x\right)^5\right]=$ This is the derivative of a function raised to a power.

$$\frac{d}{dx}\left[\left(8x^3+12x^2+6x\right)^5\right]=\underbrace{5\left(8x^3+12x^2+6x\right)^4}_{\substack{\text{power rule}\\ \text{as usual}}}\cdot\underbrace{\left(24x^2+24x+6\right)}_{\substack{\text{derivative}\\ \text{of inner}}}$$

i.e., $\frac{d}{dx}\left[\left(8x^3+12x^2+6x\right)^5\right]=5\left(8x^3+12x^2+6x\right)^4\left(24x^2+24x+6\right)$

5. Given that $f(x)=4x^3+4x-5$, give the *equation* of the line tangent to the graph of $f(x)$ at the point $(1,3)$.

We need two things:

   i. A **point** on the line (We have that: $(x_1,y_1)=(1,3)$)

   ii. The **slope** of the line (This is $f'(x_1)$)

   $f'(x)=12x^2+4$

At the point $(x_1,y_1)=(1,3)$, **the slope is** $f'(1)=12(1)^2+4=16$

We will use the Point-Slope equation of a line:

$(y-y_1)=m(x-x_1)$ (Where $m$ is the slope and $(x_1,y_1)$ is a known point on the line.)

Thus, the equation of the line tangent to the graph of $f(x)$ is:

$(y-3)=16(x-1)$

The equation of the line tangent is $(y-3)=16(x-1)$

6. Given that $w = 3u^4 + 4u$ and that $u = \cos(v)$; compute $\frac{dw}{dv}$ **using the Liebniz form of the Chain Rule.** (In particular, when doing this exercise, *write explicitly the Liebniz form of the chain rule that you are going to use.)*

   **We know:**

   $\frac{dw}{du} = 12u^3 + 4$

   $\frac{du}{dv} = -\sin(v)$

   **We want:** $\frac{dw}{dv}$

   By the Liebniz form of the Chain Rule:

   $$\frac{dw}{dv} = \frac{dw}{du}\frac{du}{dv} = (12u^3 + 4)(-\sin(v)) = \underbrace{-\left(12(\cos(v))^3 + 4\right)\sin(v)}_{\text{express solely in terms of independent variable } v}$$

   $$\boxed{\text{i.e. } \frac{dw}{dv} = -\left(12(\cos(v))^3 + 4\right)\sin(v) = -\left(12\cos^3(v) + 4\right)\sin(v)}$$

7. Compute: $\frac{d}{dx}\left[\tan\left(5x^3 + 3x^2\right)\right] =$

| | | |
|---|---|---|
| Outer: | $=$ | $\tan(\quad)$ |
| Deriv. of outer | $=$ | $\sec^2(\quad)$ |

$$\frac{d}{dx}\left[\underset{\substack{\uparrow\\ \text{outer}}}{\tan}\Big(\underset{\substack{\uparrow\\ \text{inner}}}{\underbrace{5x^3 + 3x^2}}\Big)\right] = \underbrace{\sec^2\left(5x^3 + 3x^2\right)}_{\text{Deriv of outer, eval. at inner}} \cdot \underbrace{\left(15x^2 + 6x\right)}_{\text{deriv. of inner}}$$

$$\boxed{\text{i.e., } \frac{d}{dx}\left[\tan\left(5x^3 + 3x^2\right)\right] = \sec^2\left(5x^3 + 3x^2\right)\left(15x^2 + 6x\right)}$$

8. Compute: $\frac{d}{dx}\left[\left(\frac{5x^2+12x}{8x^2+10x+1}\right)^6\right] =$ In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE.

$$\frac{d}{dx}\left[\underbrace{\left(\frac{5x^2+12x}{8x^2+10x+1}\right)^6}_{(g(x))^n}\right] = \underbrace{6\left(\frac{5x^2+12x}{8x^2+10x+1}\right)^5}_{\text{power rule as usual}} \cdot \underbrace{\left(\frac{d}{dx}\left[\frac{5x^2+12x}{8x^2+10x+1}\right]\right)}_{\text{deriv of inner Function}}$$

$$= 6\left(\frac{5x^2+12x}{8x^2+10x+1}\right)^5 \underbrace{\frac{(10x+12)\left(8x^2+10x+1\right) - (16x+10)\left(5x^2+12x\right)}{\left(8x^2+10x+1\right)^2}}_{\text{quotient rule}}$$

i.e., $\frac{d}{dx}\left[\left(\frac{5x^2+12x}{8x^2+10x+1}\right)^6\right] = 6\left(\frac{5x^2+12x}{8x^2+10x+1}\right)^5 \frac{(10x+12)\left(8x^2+10x+1\right) - (16x+10)\left(5x^2+12x\right)}{\left(8x^2+10x+1\right)^2}$

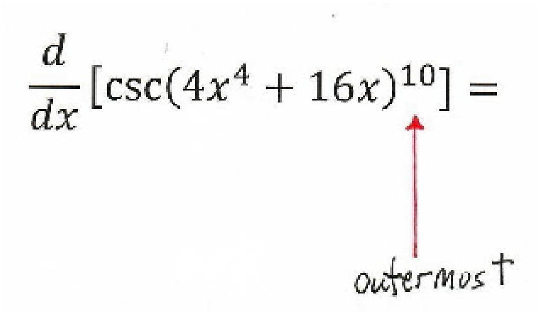
9. Compute: $\frac{d}{dx}\left[\csc^{10}\left(4x^4+16x\right)\right]=$

Let's rewrite this:

$\frac{d}{dx}\left[\left(\csc\left(4x^4+16x\right)\right)^{10}\right]$

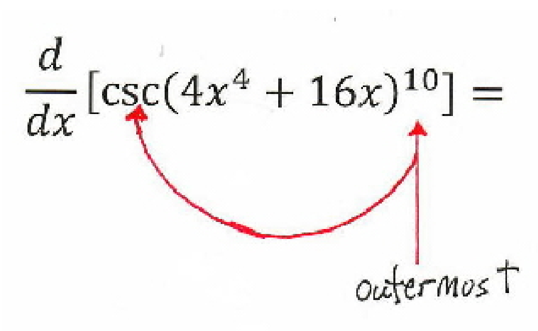
This is the composition of *three* functions.

Differentiate the outermost function and evaluate it at everything inside



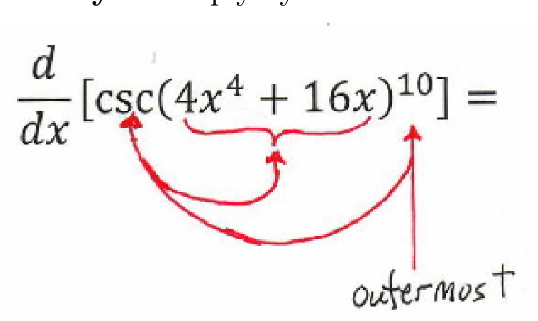

This yields: $10\left(\csc\left(4x^4+16x\right)\right)^9$

**Next:** Multiply by the derivative of the next outermost function and evaluate it at everything inside of it.




This yields: $10\left(\csc\left(4x^4+16x\right)\right)^9\cdot\left(-\csc\left(4x^4+16x\right)\cot\left(4x^4+16x\right)\right)$

**Finally:** Multiply by the derivative of the innermost function.




This yields: $10\left(\csc\left(4x^4+16x\right)\right)^9\cdot\left(-\csc\left(4x^4+16x\right)\cot\left(4x^4+16x\right)\right)\cdot\left(16x^3+16\right)$

i.e., $\frac{d}{dx}\left[\csc^{10}\left(4x^4+16x\right)\right] = -10\left(\csc\left(4x^4+16x\right)\right)^9 \cdot \csc\left(4x^4+16x\right)\cot\left(4x^4+16x\right)\cdot\left(16x^3+16\right)$

**Alternatively:**

$\frac{d}{dx}\left[\csc^{10}\left(4x^4+16x\right)\right]$ In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE. Re-write!

$\frac{d}{dx}\left[\left(\csc\left(4x^4+16x\right)\right)^{10}\right]$ This is the derivative of a function, raised to a power

$$\frac{d}{dx}\left[\left(\csc\left(4x^4+16x\right)\right)^{10}\right] = \underbrace{10\left(\csc\left(4x^4+16x\right)\right)^9}_{\substack{\text{power rule}\\ \text{as usual}}} \cdot \underbrace{\left(\frac{d}{dx}\left[\csc\left(4x^4+16x\right)\right]\right)}_{\substack{\text{derivative}\\ \text{of inner}}}$$

$$= 10\left(\csc\left(4x^4+16x\right)\right)^9 \cdot \underbrace{\left[-\csc\left(4x^4+16x\right)\cot\left(4x^4+16x\right)\cdot\left(16x^3+16\right)\right]}_{\substack{\text{Chain}\\ \text{Rule}}}$$

i.e., $\frac{d}{dx}\left[\csc^{10}\left(4x^4+16x\right)\right] = -10\left(\csc\left(4x^4+16x\right)\right)^9 \csc\left(4x^4+16x\right)\cot\left(4x^4+16x\right)\cdot\left(16x^3+16\right)$

10. Given that $x^6 - x^6y^4 = \sin(y)$, compute $\frac{dy}{dx}$

i. Differentiate both sides w.r.t. $x$

$$\frac{d}{dx}\left[x^6 - \underbrace{x^6}_{1^{\text{st}}}\underbrace{y^4}_{2^{\text{nd}}}\right] = \frac{d}{dx}\left[\sin(y)\right]$$

$$\Rightarrow 6x^5 - \left(\underbrace{6x^5}_{1^{\text{st}}\text{ prime}} \cdot \underbrace{y^4}_{2^{\text{nd}}} + \underbrace{4y^3\frac{dy}{dx}}_{2^{\text{nd}}\text{ prime}} \cdot \underbrace{x^6}_{1^{\text{st}}}\right) = \cos(y) \cdot \frac{dy}{dx}$$

Simplifying, we have:

$$6x^5 - 6x^5y^4 - 4x^6y^3\frac{dy}{dx} = \cos(y)\frac{dy}{dx}$$

ii. Solve algebraically for $\frac{dy}{dx}$

a. Get $\frac{dy}{dx}$ terms on left side, all other terms on right side

$$\Rightarrow -4x^6y^3\frac{dy}{dx} - \cos(y)\frac{dy}{dx} = -6x^5 + 6x^5y^4$$

b. Factor out $\frac{dy}{dx}$

$$\Rightarrow \frac{dy}{dx}\left(-4x^6y^3 - \cos(y)\right) = -6x^5 + 6x^5y^4$$

c. Divide both sides by the cofactor of $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-6x^5+6x^5y^4}{-4x^6y^3-\cos(y)} = \frac{6x^5-6x^5y^4}{4x^6y^3+\cos(y)}$$

$$\boxed{\frac{dy}{dx} = \frac{-6x^5+6x^5y^4}{-4x^6y^3-\cos(y)} = \frac{6x^5-6x^5y^4}{4x^6y^3+\cos(y)}}$$

11. Given that $f(x) = 3x^2 - 9x + 2$, compute $f'(x)$ **using the definition of derivative.** (i.e., using the "limit process.")

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[3(x+\Delta x)^2 - 9(x+\Delta x) + 2\right] - \left[3x^2 - 9x + 2\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[3\left(x^2 + 2x\Delta x + \Delta x^2\right) - 9(x+\Delta x) + 2\right] - \left[3x^2 - 9x + 2\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[3x^2 + 6x\Delta x + 3\Delta x^2 - 9x - 9\Delta x + 2\right] - \left[3x^2 - 9x + 2\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{6x\Delta x + 3\Delta x^2 - 9\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(6x + 3\Delta x - 9)}{\Delta x} = \lim_{\Delta x \to 0} (6x + 3\Delta x - 9) = 6x + 3(0) - 9 = 6x - 9$$

$$\boxed{\text{i.e., } f'(x) = 6x - 9}$$

**Extra** (Wow! 10 Points)

Given that $S'(x) = \frac{1}{\sqrt{1-x^2}}$ (i.e., $\frac{d}{dx}[S(x)] = \frac{1}{\sqrt{1-x^2}}$); compute $\frac{d}{dx}[S(\sin(x))]$

| | | |
|---|---|---|
| Outer: | $=$ | $S(\quad)$ |
| Deriv. of outer | $=$ | $\frac{1}{\sqrt{1-(\quad)^2}}$ |

$$\frac{d}{dx}\Bigg[\underset{\uparrow \atop \text{outer}}{S}\Bigg(\underset{\uparrow \atop \text{inner}}{\underbrace{\sin(x)}}\Bigg)\Bigg] = \underbrace{\frac{1}{\sqrt{1-(\sin(x))^2}}}_{\text{Deriv of outer, eval. at inner}} \cdot \underbrace{\cos(x)}_{\text{deriv. of inner}} = \frac{\cos(x)}{\sqrt{1-\sin^2(x)}}$$

$$\boxed{\text{i.e., } \frac{d}{dx}[S(\sin(x))] = \frac{\cos(x)}{\sqrt{1-\sin^2(x)}}}$$