MTH 1125 2pm Class - Test #4 - Solutions

Fall 2018

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Show CLEARLY how you arrive at your answers!

1. **Compute:** $\int \left(16x^3 + 12x^2 + 8x + 4 + 2\sqrt{x}\right) dx =$

$\int \left(16x^3 + 12x^2 + 8x + 4 + 2\sqrt{x}\right) dx = \int \left(16x^3 + 12x^2 + 8x + 4 + 2x^{\frac{1}{2}}\right) dx$

$= 16\left[\frac{x^4}{4}\right] + 12\left[\frac{x^3}{3}\right] + 8\left[\frac{x^2}{2}\right] + 4x + 2\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C = 4x^4 + 4x^3 + 4x^2 + 4x + 2\left(\frac{2}{3}\right)x^{\frac{3}{2}} + C$

$= 4x^4 + 4x^3 + 4x^2 + 4x + \frac{4}{3}x^{\frac{3}{2}} + C$

i.e., $\int \left(16x^3 + 12x^2 + 8x + 4 + 2\sqrt{x}\right) dx = 4x^4 + 4x^3 + 4x^2 + 4x + \frac{4}{3}x^{\frac{3}{2}} + C$

2. **Compute:** $\int \left(9x^2+12x\right)^5 (3x+2)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(9x^2+12x\right)^5$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(9x^2+12x\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(9x^2+12x\right)}_{\text{function}} - - - - \rightarrow \underbrace{(3x+2)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(9x^2+12x\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 9x^2+12x \\ \Rightarrow \frac{du}{dx} &= 18x+12 \\ \Rightarrow du &= (18x+12)\,dx \\ \Rightarrow \frac{1}{6}du &= (3x+2)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\left(9x^2+12x\right)^5}_{u^5}\underbrace{(3x+2)\,dx}_{\frac{1}{6}du} = \int u^5 \tfrac{1}{6}du = \tfrac{1}{6}\int u^5 du$$

4. Integrate (in terms of u).

$$\tfrac{1}{6}\int u^5 du = \tfrac{1}{6}\left[\tfrac{u^6}{6}\right] + C = \tfrac{1}{36}u^6 + C$$

5. Re-express in terms of the original variable, x.

$$\int \left(9x^2+12x\right)^5 (3x+2)\,dx = \underbrace{\frac{1}{36}\left(9x^2+12x\right)^6 + C}_{\frac{1}{36}u^6+C}$$

$$\text{i.e., } \int \left(9x^2+12x\right)^5 (3x+2)\,dx = \tfrac{1}{36}\left(9x^2+12x\right)^6 + C$$

3. **Compute:** $\int \left(5\cos(x) + 6\csc^2(x)\right) dx =$

$\int \left(5\cos(x) + 6\csc^2(x)\right) dx = 5\left[\sin(x)\right] + 6\left[-\cot(x)\right] + C$

$= 5\sin(x) - 6\cot(x) + C$

i.e., $\int \left(5\cos(x) + 6\csc^2(x)\right) dx = 5\sin(x) - 6\cot(x) + C$

4. **Compute:** $\int \sin\left(6x^2+18x+4\right)(2x+3)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\underset{\text{outer}}{\sin}\left(\underset{\text{inner}}{6x^2+18x+4}\right)$

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(6x^2+18x+4\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(6x^2+18x+4\right)}_{\text{function}} - - - - \rightarrow \underbrace{(2x+3)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(3x^5+5x\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 6x^2+18x+4 \\ \Rightarrow \frac{du}{dx} &= 12x+18 \\ \Rightarrow du &= (12x+18)\,dx \\ \Rightarrow \frac{1}{6}du &= (2x+3)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\sin\left(6x^2+18x+4\right)}_{\sin(u)}\underbrace{(2x+3)\,dx}_{\frac{1}{6}du} = \int \sin(u)\ \frac{1}{6}du = \frac{1}{6}\int \sin(u)\,du$$

4. Integrate (in terms of u).

$$\frac{1}{6}\int \sin(u)\,du = \frac{1}{6}\left[-\cos(u)\right] + C = -\frac{1}{6}\cos(u) + C$$

5. Re-express in terms of the original variable, x.

$$\int \sin\left(6x^2+18x+4\right)(2x+3)\,dx = \underbrace{-\frac{1}{6}\cos\left(6x^2+18x+4\right)+C}_{-\frac{1}{6}\cos(u)+C}$$

i.e., $\int \sin\left(6x^2+18x+4\right)(2x+3)\,dx = -\frac{1}{6}\cos\left(6x^2+18x+4\right)+C$

5. **Compute:** $\int_{-1}^{1}\left(6x^2+6x+4\right)dx =$

$$\int_{-1}^{1}\underbrace{\left(6x^2+6x+4\right)}_{f(x)}dx = \underbrace{\left[6\frac{x^3}{3}+6\frac{x^2}{2}+4x\right]_{-1}^{1}}_{F(x)} = \underbrace{\left[2x^3+3x^2+4x\right]_{-1}^{1}}_{F(x)} =$$

$$= \underbrace{\left[2\left(1\right)^3+3\left(1\right)^2+4\left(1\right)\right]}_{F(1)} - \underbrace{\left[2\left(-1\right)^3+3\left(-1\right)^2+4\left(-1\right)\right]}_{F(-1)} = 12$$

i.e., $\int_{-1}^{1}\left(6x^2+6x+4\right)dx = 12$

6. **Compute:** $\int_0^1 \left(4x^2+2\right)^3 x\ dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(4x^2+2\right)^3$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(4x^2+2\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(4x^2+2\right)}_{\text{function}} - - - - - \rightarrow \underbrace{x}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(4x^2+2\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{array}{rcl} u & = & 4x^2+2 \\ \Rightarrow \frac{du}{dx} & = & 8x \\ \Rightarrow du & = & 8xdx \\ \Rightarrow \frac{1}{8}du & = & x\ dx \end{array}$$

   When $x = 0$, $u = 4x^2+2 = 4\left(0\right)^2+2 = 2$
   When $x = 1$, $u = 4x^2+2 = 4\left(1\right)^2+2 = 6$

3. Analyze in terms of u and du

$$\int_{x=0}^{x=1} \underbrace{\left(4x^2+2\right)^3}_{u^3} \underbrace{x\ dx}_{\frac{1}{8}du} = \int_{u=2}^{u=6} u^3 \cdot \tfrac{1}{8}du = \tfrac{1}{8}\int_{u=2}^{u=6} u^3 du$$

   Don't forget to re-write the limits of integration in terms of u!

4. Integrate (in terms of u).

$$\tfrac{1}{8}\int_{u=2}^{u=6} u^3 du = \tfrac{1}{8}\left[\tfrac{u^4}{4}\right]_{u=2}^{u=6} = \left[\tfrac{u^4}{32}\right]_{u=2}^{u=6} = \underbrace{\frac{(6)^4}{32}}_{F(6)} - \underbrace{\frac{(2)^4}{32}}_{F(2)} = \tfrac{81}{2} - \tfrac{1}{2} = 40$$

$$\text{i.e., } \int_{x=0}^{x=1} \left(4x^2+2\right)^3 x\ dx = 40$$

7. **Compute:** $\frac{d}{dx}\left[\ln\left(\sqrt{x}\right)\right] \underbrace{=}_{\text{re-write}} \frac{d}{dx}\left[\ln\left(x^{\frac{1}{2}}\right)\right] \underbrace{=}_{\text{re-write}} \frac{d}{dx}\left[\frac{1}{2}\ln\left(x\right)\right] \underbrace{=}_{\text{re-write}} \frac{1}{2}\frac{d}{dx}\left[\ln\left(x\right)\right]$

$$\frac{1}{2}\underbrace{\frac{d}{dx}\left[\ln\left(x\right)\right]}_{\frac{d}{dx}[\ln(g(x))]} = \frac{1}{2}\underbrace{\frac{1}{x}}_{\frac{1}{g(x)}} \cdot \underbrace{1}_{g'(x)} = \frac{1}{2}x$$

i.e., $\frac{d}{dx}\left[\ln\left(\sqrt{x}\right)\right] = \frac{1}{2}x$

8. **Compute:** $\int \frac{\sec^2(3x)}{\tan(3x)} dx =$

$$\int \frac{\sec^2(3x)}{\tan(3x)} dx \underbrace{=}_{\text{re-write}} \int \frac{1}{\tan(3x)} \sec^2(3x)\ dx$$

Remark: Note that we have an approximate function/derivative pair, with the "function" in the denominator. This usually indicates that u-substitution is appropriate, with the result being a natural logarithm.

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\frac{1}{\tan(3x)}$ is the same as $(\tan(3x))^{-1}$, so it is a function raised to a power.

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \tan(3x)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{(\tan(3x))}_{\text{function}} - - - - \rightarrow \underbrace{\sec^2(3x)\ dx}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \tan(3x)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= \tan(3x) \\ \Rightarrow \frac{du}{dx} &= 3\sec^2(3x) \\ \Rightarrow du &= 3\sec^2(3x)\,dx \\ \Rightarrow \frac{1}{3}du &= \sec^2(3x)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \frac{\sec^2(3x)}{\tan(3x)} dx \underbrace{\int \frac{1}{\tan(3x)}}_{\frac{1}{u}} \underbrace{\sec^2(3x)\ dx}_{\frac{1}{3}du} = \int \frac{1}{u}\frac{1}{3}du = \frac{1}{3}\int \frac{1}{u}du$$

4. Integrate (in terms of u).

$$\frac{1}{3}\int \frac{1}{u}du = \frac{1}{3}\ln|u| + C$$

5. Re-express in terms of the original variable, x.

$$\int \frac{\sec^2(3x)}{\tan(3x)} dx = \underbrace{\frac{1}{3}\ln|\tan(3x)| + C}_{\frac{1}{3}\ln|u|+C}$$

$$\text{i.e., } \int \frac{\sec^2(3x)}{\tan(3x)} dx = \frac{1}{3} \ln|\tan(3x)| + C$$