Mth 123 Test #1 - Solutions

Fall 1996

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Instructions. Show clearly how you arrive at your answers.

1. Compute: $\lim_{x\to 2} \frac{x^3-2x^2+2}{x^3+3} =$

   (a) 1. Try plugging in:

   $\lim_{x\to 2} \frac{x^3-2x^2+2}{x^3+3} = \frac{2^3-2(2)^2+2}{2^3+3} = \frac{2}{11}$

   i.e., $\lim_{x\to 2} \frac{x^3-2x^2+2}{x^3+3} = \frac{2}{11}$

2. Compute: $\lim_{x\to 2} \frac{x^2-4}{x^2+2x-8} =$

   (a) 1. Try plugging in:

   $\lim_{x\to 2} \frac{x^2-4}{x^2+2x-8} = \frac{2^2-4}{2^2+2(2)-8} = \frac{0}{0}$ No Good! Zero Divide

   2. Try factoring out the "zero factor" from top and bottom:

   $\lim_{x\to 2} \frac{x^2-4}{x^2+2x-8} = \lim_{x\to 2} \frac{(x+2)(x-2)}{(x+4)(x-2)} = \lim_{x\to 2} \frac{(x+2)}{(x+4)} \underbrace{=}_{\text{NOW plug in!}} \frac{((2)+2)}{((2)+4)} = \frac{2}{3}$

   i.e., $\lim_{x\to 2} \frac{x^2-4}{x^2+2x-8} = \frac{2}{3}$

3. Compute: $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x} =$

   (a) 1. Try plugging in:

   $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x} = \frac{\sqrt{3+0}-\sqrt{3}}{0} = \frac{0}{0}$ No Good! Zero Divide

   2. Try factoring out the "zero factor" from top and bottom:

   $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x} = \lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x} \cdot \frac{\sqrt{3+x}+\sqrt{3}}{\sqrt{3+x}+\sqrt{3}} = \lim_{x\to 0} \frac{\left(\sqrt{3+x}\right)^2-\left(\sqrt{3}\right)^2}{x\left(\sqrt{3+x}+\sqrt{3}\right)}$

   $= \lim_{x\to 0} \frac{3+x-3}{x\left(\sqrt{3+x}+\sqrt{3}\right)} = \lim_{x\to 0} \frac{x}{x\left(\sqrt{3+x}+\sqrt{3}\right)}$

   $= \lim_{x\to 0} \frac{1}{\left(\sqrt{3+x}+\sqrt{3}\right)} \underbrace{=}_{\text{NOW plug in!}} \frac{1}{\left(\sqrt{3+0}+\sqrt{3}\right)} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

   i.e., $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

4. $\lim_{x\to\infty} \frac{2x^3-3x+4}{6x^3-6x^2+5x-2} =$

   Observe: as $x \to \infty$, the terms of highest degree dominate the numerator and denominator. Consequently,

   $\lim_{x\to\infty} \frac{2x^3-3x+4}{6x^3-6x^2+5x-2} = \lim_{x\to\infty} \frac{2x^3}{6x^3} = \lim_{x\to\infty} \frac{1}{3} = \frac{1}{3}$

   i.e., $\lim_{x\to\infty} \frac{2x^3-3x+4}{6x^3-6x^2+5x-2} = \frac{1}{3}$

5. Find asymptotes and graph: $f(x) = \frac{2x+5}{x-3}$

Verticals: Look for x values that cause division by zero.

   We have division by zero, exactly when $x - 3 = 0$

   $\Rightarrow x = 3$

   Next, we examine the one-sided limits at $x = 3$.

   $\lim_{x\to 3^-} \frac{2x+5}{x-3} = \frac{11}{-\varepsilon} = -\infty$

   $x \to 3^-$
   $\Rightarrow \quad x < 3$
   $\Rightarrow \quad x - 3 < 0$

   $\lim_{x\to 3^+} \frac{2x+5}{x-3} = \frac{11}{\varepsilon} = +\infty$

   $x \to 3^+$
   $\Rightarrow \quad x > 3$
   $\Rightarrow \quad x - 3 > 0$

   i.e., $\lim_{x\to 3^-} \frac{2x+5}{x-3} = -\infty \leftarrow$

   Infinite limits tell us that
   $x = 3$ IS a vertical asymptote

   $\swarrow$

   and $\lim_{x\to 3^+} \frac{2x+5}{x-3} = +\infty$

Horizontals: Compute limits as $x \to \pm\infty$

   Observe: as $x \to \infty$, the terms of highest degree dominate the numerator and denominator. Consequently,

   $\lim_{x\to -\infty} \frac{2x+5}{x-3} = \lim_{x\to -\infty} \frac{2x}{x} = \lim_{x\to -\infty} 2 = 2$

   $\lim_{x\to +\infty} \frac{2x+5}{x-3} = \lim_{x\to +\infty} \frac{2x}{x} = \lim_{x\to +\infty} 2 = 2$

i.e., $\lim_{x\to-\infty}\frac{2x+5}{x-3}=2\leftarrow$

Finite limits tell us that
$x=2$ IS a horizontal asymptote

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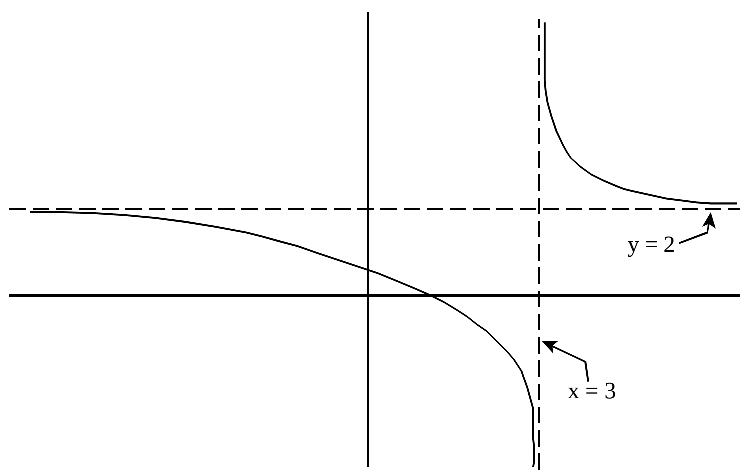
$\lim_{x\to+\infty}\frac{2x+5}{x-3}=2$

Graph: $f(x)=\frac{2x+5}{x-3}$



6. $\lim_{x\to-\infty}\frac{3x^3+2x+5}{9x^2+4x-2}=$

   Observe: as $x\to-\infty$, the terms of highest degree dominate the numerator and denominator. Consequently,
   $\lim_{x\to-\infty}\frac{3x^3+2x+5}{9x^2+4x-2}=\lim_{x\to-\infty}\frac{3x^3}{9x^2}=\lim_{x\to-\infty}\frac{x}{3}=-\infty$

   i.e., $\lim_{x\to-\infty}\frac{3x^3+2x+5}{9x^2+4x-2}=-\infty$

7. $\lim_{x\to2}\frac{x^2+1}{x^2-x-2}=$

   (a) 1. Try plugging in:

   $\lim_{x\to2}\frac{x^2+1}{x^2-x-2}=\frac{2^2+1}{2^2-2-2}=\frac{5}{0}$ No Good! Zero Divide

   2. Try factoring out the "zero factor" from top and bottom:

   Since step #1 didn't yield an expression of the form: $\frac{0}{0}$, step #2 won't work.

3. Examnine the one sided limits, as $x \to 2$

   $\lim_{x\to 2^-} \frac{x^2+1}{x^2-x-2} = \lim_{x\to 2^-} \frac{x^2+1}{(x+1)(x-2)} = \frac{5}{(3)(-\varepsilon)} = \frac{\left(\frac{5}{3}\right)}{-\varepsilon} = -\infty$

   $x \to 2^-$
   $\Rightarrow \quad x < 2$
   $\Rightarrow \quad x - 2 < 0$

   $\lim_{x\to 2^+} \frac{x^2+1}{x^2-x-2} = \lim_{x\to 2^+} \frac{x^2+1}{(x+1)(x-2)} = \frac{5}{(3)\varepsilon} = \frac{\left(\frac{5}{3}\right)}{\varepsilon} = +\infty$

   $x \to 2^+$
   $\Rightarrow \quad x > 2$
   $\Rightarrow \quad x - 2 > 0$

   i.e., $\lim_{x\to 2^-} \frac{x^2+1}{x^2-x-2} = -\infty \leftarrow$

   Since the one-sided limits are not equal, the limit DOES NOT EXIST.

   $\swarrow$

   and $\lim_{x\to 2^+} \frac{x^2+1}{x^2-x-2} = +\infty$

8. Given:

$x =$	$f(x)$
4.000	-3.5
4.500	-35.1
4.900	-351.2
4.990	-3512.3
4.999	-35123.0

$x =$	$f(x)$
6.000	-3.5
5.500	-35.1
5.100	-351.2
5.010	-3512.3
5.001	-35123.0

   determine:

   (a) $\lim_{x\to 5^-} f(x) =$

   Note that as $x \to 5^-$ (in the first table), $f(x)$ is negative, and becomes "large without bound." Hence:

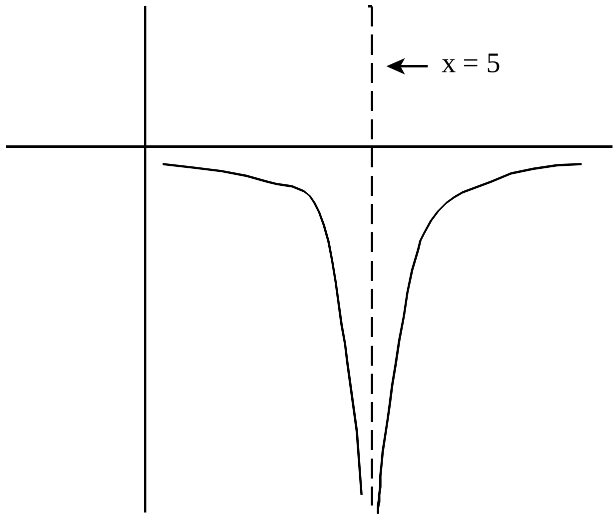
   $$\lim_{x\to 5^-} f(x) = -\infty$$

   (b) $\lim_{x\to 5^+} f(x) =$

   Note that as $x \to 5^+$ (in the second table), $f(x)$ is negative, and becomes "large without bound." Hence:

   $$\lim_{x\to 5^+} f(x) = -\infty$$

(c) Sketch a rough graph of $f(x)$.



9. Given:

$x =$	$f(x)$
-10.0	-1.5601
-100.0	-1.1311
-1,000.0	-1.0132
-10,000.0	-1.0012
-100,000.0	-1.0002

$x =$	$f(x)$
10.0	-0.4399
100.0	-0.8689
1,000.0	-0.9868
10,000.0	-0.9988
100,000.0	-0.9998

determine:

(a) $\lim_{x\to-\infty} f(x) =$

Note that as $x \to -\infty$ (in the first table), $f(x)$ approaches -1^-. Hence:

$$\lim_{x\to-\infty} f(x) = -1$$

(b) $\lim_{x\to+\infty} f(x) =$

Note that as $x \to \infty$ (in the second table), $f(x)$ approaches -1^+. Hence:

$$\lim_{x\to\infty} f(x) = -1$$

(c) Sketch a rough graph of $f(x)$.