MTH 1126 Practice Test #1_2 - Solutions

Spring 2017

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Instructions

Answers appear on the ANSWERS page. Solutions appear on the SOLUTIONS page.

1. Compute: $\int \left(2x^4 + 6x^3 + 3x + 6\sqrt{x} + 2\right) dx =$

   (Re-write) $\int \left(2x^4 + 6x^3 + 3x + 6x^{\frac{1}{2}} + 2\right) dx = 2\left[\frac{x^5}{5}\right] + 6\left[\frac{x^4}{4}\right] + 3\left[\frac{x^2}{2}\right] + 6\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right] + 2x + C$

   $= \frac{2}{5}x^5 + \frac{3}{2}x^4 + \frac{3}{2}x^2 + 4x^{\frac{3}{2}} + 2x + C$

   i.e., $\int \left(2x^4 + 6x^3 + 3x + 6\sqrt{x} + 2\right) dx = \frac{2}{5}x^5 + \frac{3}{2}x^4 + \frac{3}{2}x^2 + 4x^{\frac{3}{2}} + 2x + C$ (Don't forget the "+C")

2. Compute: $\int \left(2\cos(x) + 5\sec^2(x)\right) dx =$

   $\int \left(2\cos(x) + 5\sec^2(x)\right) dx = 2[\sin(x)] + 5[\tan(x)] + C$

   i.e., $\int \left(2\cos(x) + 5\sec^2(x)\right) dx = 2[\sin(x)] + 5[\tan(x)] + C$ (Don't forget the "+C")

3. Compute: $\int_{x=0}^{x=2} \left(2x^3 + 3x^2 + 2\right) dx =$

   $$\int_{x=0}^{x=2} \underbrace{\left(2x^3 + 3x^2 + 2\right)}_{f(x)} dx = \underbrace{\left[\frac{1}{2}x^4 + x^3 + 2x\right]_{x=0}^{x=2}}_{F(x)}$$

   $$= \underbrace{\left[\frac{1}{2}(2)^4 + (2)^3 + 2(2)\right]}_{F(2)} - \underbrace{\left[\frac{1}{2}(0)^4 + (0)^3 + 2(0)\right]}_{F(0)} = 20$$

   i.e., $\int_{x=0}^{x=2} \left(2x^3 + 3x^2 + 2\right) dx = 20$

4. Compute: $\int \left(2x^3+2x\right)^5 \left(3x^2+1\right) dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

      Yes! $\left(2x^3+2x\right)^5$ (A function raised to a power is always a composite function!)

      Let $u =$ the "inner" of the composite function

      $\Rightarrow u = \left(2x^3+2x\right)$

   b. Is there an (approximate) function/derivative pair?

      Yes! $\underbrace{\left(2x^3+2x\right)}_{\text{function}} - - - - \rightarrow \underbrace{\left(3x^2+1\right)}_{\text{deriv}}$

      Let $u =$ the "function" of the function/deriv pair

      $\Rightarrow u = \left(2x^3+2x\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

      (i.e., do criteria **a** and **b** suggest the same choice of u ?)

      Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{array}{rcl} u & = & 2x^3+2x \\ \Rightarrow \frac{du}{dx} & = & 6x^2+2 \\ \Rightarrow du & = & \left(6x^2+2\right) dx \\ \Rightarrow \frac{1}{2} du & = & \left(3x^2+1\right) dx \end{array}$$

3. Analyze in terms of u and du

$$\int \underbrace{\left(2x^3+2x\right)^5}_{u^5} \underbrace{\left(3x^2+1\right) dx}_{\frac{1}{2}du} = \int u^5 \tfrac{1}{2} du = \tfrac{1}{2} \int u^5 du$$

4. Integrate (in terms of u).

$$\tfrac{1}{2} \int u^5 du = \tfrac{1}{2} \left[\tfrac{u^6}{6}\right] + C = \tfrac{1}{12} u^6 + C$$

5. Re-express in terms of the original variable, x.

$$\int \left(2x^3+2x\right)^5 \left(3x^2+1\right) dx = \underbrace{\frac{1}{12} \left(2x^3+2x\right)^6 + C}_{\frac{1}{12}u^6 + C}$$

i.e., $\int \left(2x^3+2x\right)^5 \left(3x^2+1\right) dx = \frac{1}{12} \left(2x^3+2x\right)^6 + C$

5. Compute: $\int \cos\left(3x^2\right)\, x\, dx =$

   1. Is u-sub appropriate?

      a. Is there a composite function?

      Yes! $\cos\left(3x^2\right)$

      $\nearrow$ $\nwarrow$

      outer inner

      Let $u =$ the "inner" of the composite function

      $\Rightarrow u = 3x^2$

      b. Is there an (approximate) function/derivative pair?

      Yes! $\underbrace{3x^2}_{\text{function}} - - - - \rightarrow \underbrace{x}_{\text{deriv}}$

      Let $u =$ the "function" of the function/deriv pair

      $\Rightarrow u = 3x^2$

      c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

      (i.e., do criteria **a** and **b** suggest the same choice of u ?)

      Yes! $\Rightarrow$ u-substitution is appropriate

   2. Compute du

$$\begin{aligned} u &= 3x^2 \\ \Rightarrow \frac{du}{dx} &= 6x \\ \Rightarrow du &= 6x\, dx \\ \Rightarrow \frac{1}{6} du &= x\, dx \end{aligned}$$

   3. Analyze in terms of u and du

$$\int \underbrace{\cos\left(3x^2\right)}_{\cos(u)} \underbrace{x\, dx}_{\frac{1}{6}du} = \int \cos(u)\ \frac{1}{6}du = \frac{1}{6}\int \cos(u)\, du$$

   4. Integrate (in terms of u).

$$\frac{1}{6}\int \cos(u)\, du = \frac{1}{6}\left[\sin(u)\right] + C = \frac{1}{6}\sin(u) + C$$

   5. Re-express in terms of the original variable, x.

$$\int \cos\left(3x^2\right)\, x\, dx = \underbrace{\frac{1}{6}\sin\left(3x^2\right) + C}_{\frac{1}{6}\sin(u) + C}$$

i.e., $\int \cos\left(3x^2\right)\, x\, dx = \frac{1}{6}\sin\left(3x^2\right) + C$

6. Compute: $\int \frac{x}{3x^2+6} dx =$

$\int \frac{x}{3x^2+6} dx \underbrace{=}_{\text{re-write}} \int \frac{1}{3x^2+6}\, x\, dx$

Remark: Note that we have an approximate function/derivative pair, with the "function" in the denominator. This usually indicates that u-substitution is appropriate, with the result being a natural logarithm.

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\frac{1}{3x^2+6}$ is the same as $\left(3x^2+6\right)^{-1}$, so it is a function raised to a power.

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(3x^2+6\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(3x^2+6\right)}_{\text{function}} - - - - \rightarrow \underbrace{x}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(3x^2+6\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 3x^2+6 \\ \Rightarrow \frac{du}{dx} &= 6x \\ \Rightarrow du &= 6x dx \\ \Rightarrow \frac{1}{6} du &= x dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\frac{1}{3x^2+6}}_{\frac{1}{u}} \underbrace{x\,dx}_{\frac{1}{6}du} = \int \frac{1}{u} \cdot \frac{1}{6} du = \frac{1}{6} \int \frac{1}{u} du$$

4. Integrate (in terms of u).

$\frac{1}{6} \int \frac{1}{u} du = \frac{1}{6} \left[\ln|u|\right] + C = \frac{1}{6} \ln|u| + C$

5. Re-express in terms of the original variable, x.

$$\int \frac{x}{3x^2+6} dx = \underbrace{\frac{1}{6} \ln\left|3x^2+6\right| + C}_{\frac{1}{6}\ln|u|+C}$$

i.e., $\int \frac{x}{3x^2+6} dx = \frac{1}{6} \ln\left|3x^2+6\right| + C$

7. Compute: $\frac{d}{dx}\left[\ln\left(\cos\left(x\right)\right)\right] =$

$$\underbrace{\frac{d}{dx}\left[\ln\left(\cos\left(x\right)\right)\right]}_{\frac{d}{dx}\left[\ln(g(x))\right]} = \underbrace{\frac{1}{\cos\left(x\right)}}_{\frac{1}{g(x)}} \cdot \underbrace{\left(-\sin\left(x\right)\right)}_{g'(x)} = -\frac{\sin(x)}{\cos(x)} = -\tan\left(x\right)$$

i.e., $\frac{d}{dx}\left[\ln\left(\cos\left(x\right)\right)\right] = -\frac{\sin(x)}{\cos(x)} = -\tan\left(x\right)$

8. Compute: $\frac{d}{dx}\left[\ln\left(5x^2+5x\right)\right] =$

$$\underbrace{\frac{d}{dx}\left[\ln\left(5x^2+5x\right)\right]}_{\frac{d}{dx}\left[\ln(g(x))\right]} = \underbrace{\frac{1}{5x^2+5x}}_{\frac{1}{g(x)}} \cdot \underbrace{\left(10x+5\right)}_{g'(x)} = \frac{10x+5}{5x^2+5x}$$

i.e., $\frac{d}{dx}\left[\ln\left(5x^2+5x\right)\right] = \frac{10x+5}{5x^2+5x}$

9. Compute: $\frac{d}{dx}\left[\ln\left(x\sqrt{x^2-1}\right)\right] \underbrace{=}_{\text{re-write}} \frac{d}{dx}\left[\ln\left(x\left(x^2-1\right)^{\frac{1}{2}}\right)\right]$

Remark: We can compute this derivative directly, in it's current form, but it would be much easier to use the *algebraic properties* of natural logarithms to simplify the expression first.

$$\frac{d}{dx}\left[\ln\left(x\left(x^2-1\right)^{\frac{1}{2}}\right)\right] = \underbrace{\frac{d}{dx}\left[\ln\left(x\right)+\ln\left[\left(x^2-1\right)^{\frac{1}{2}}\right]\right]}_{\ln(ab)=\ln(a)+\ln(b)} = \underbrace{\frac{d}{dx}\left[\ln\left(x\right)+\frac{1}{2}\ln\left(x^2-1\right)\right]}_{\ln(a^n)=n\ln(a)}$$

NOW we're ready to compute the derivative!

$$\frac{d}{dx}\left[\ln\left(x\sqrt{x^2-1}\right)\right] = \frac{d}{dx}\left[\ln\left(x\right)+\frac{1}{2}\ln\left(x^2-1\right)\right] = \frac{d}{dx}\left[\ln\left(x\right)\right] + \frac{d}{dx}\left[\frac{1}{2}\ln\left(x^2-1\right)\right]$$

$$= \frac{1}{x} + \frac{1}{2}\frac{1}{x^2-1}\cdot 2x = \frac{1}{x} + \frac{x}{x^2-1}$$

i.e., $\frac{d}{dx}\left[\ln\left(x\sqrt{x^2-1}\right)\right] = \frac{1}{x} + \frac{x}{x^2-1}$

10. Compute: $\int_{x=0}^{x=1} \left(x^2+1\right)^3 x\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(x^2+1\right)^3$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(x^2+1\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(x^2+1\right)}_{\text{function}} - - - - - \rightarrow \underbrace{x}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(x^2+1\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= x^2+1 \\ \Rightarrow \frac{du}{dx} &= 2x \\ \Rightarrow du &= 2xdx \\ \Rightarrow \frac{1}{2}du &= xdx \end{aligned}$$

When $x=0$, $u = x^2+1 = (0)^2+1 = 1$
When $x=1$, $u = x^2+1 = (1)^2+1 = 2$

3. Analyze in terms of u and du

$$\int_{x=0}^{x=1} \underbrace{\left(x^2+1\right)^3}_{u^3} \underbrace{x\,dx}_{\frac{1}{2}du} = \int_{u=1}^{u=2} u^3 \cdot \frac{1}{2}du = \frac{1}{2}\int_{u=1}^{u=2} u^3 du$$

Don't forget to re-write the limits of integration in terms of u!

4. Integrate (in terms of u).

$$\frac{1}{2}\int_{u=1}^{u=2} u^3 du = \frac{1}{2}\left[\frac{u^4}{4}\right]_{u=1}^{u=2} = \frac{1}{8}\left[u^4\right]_{u=1}^{u=2} = \frac{1}{8}\left(\underbrace{(2)^4}_{F(2)} - \underbrace{(1)^4}_{F(1)}\right) = \frac{15}{8}$$

i.e., $\int_{x=0}^{x=1} \left(x^2+1\right)^3 x\,dx = \frac{15}{8}$

11. Write the given equation in algebraic form.

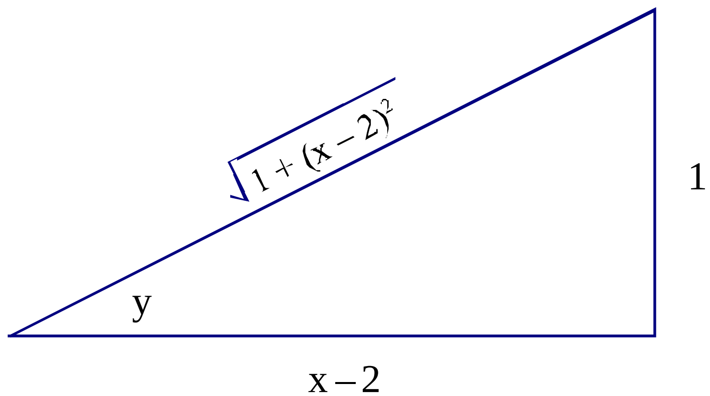
$z = \sin(\operatorname{arccot}(x-2))$

Let $y = \operatorname{arccot}(x-2)$

This is the same as saying "y is the angle whose cotangent is $x-2$."

i.e., $\cot(y) = x-2$.

Let's draw a triangle to depict this relationship.



By Pythagorean's Theorem, $(opp)^2 + (adj)^2 = (hyp)^2$

$\Rightarrow (hyp) = \sqrt{(opp)^2 + (adj)^2} = \sqrt{1+(x-2)^2}$.

Recall: we want $z = \sin(\operatorname{arccot}(x-2)) = \sin(y)$

From the picture, $z = \sin(y) = \frac{opp}{hyp} = \frac{1}{\sqrt{1+(x-2)^2}}$

Hence, $z = \sin(\operatorname{arccot}(x-2)) = \frac{1}{\sqrt{1+(x-2)^2}}$

i.e., $z = \frac{1}{\sqrt{1+(x-2)^2}}$

12. Compute: $\int \sec^3(x)\tan(x)\,dx \underset{\text{re-write}}{=} \int (\sec(x))^2 \sec(x)\tan(x)\,dx$

Remark: Notice what we did here, and WHY we did it. We removed a factor of $\sec(x)$ from $\sec^3(x)$ so that $\sec(x)\tan(x)\,dx$ can serve as the "du" of $u = \sec(x)$.

(a) 1. U-sub appropriate?

1. Composite function? Yes!

   $(\sec x)^2$

   Let $u = \sec(x)$ (The "inner" function)

2. Approx funct/deriv pair? Yes!

   $\underbrace{\sec(x)}_{\text{function}} \rightarrow \underbrace{\sec(x)\tan(x)}_{\text{deriv.}}$

   Let $u = \sec(x)$ (The "function")

2. Compute du

   $u = \sec(x)$

   $\Rightarrow \frac{du}{dx} = \sec(x)\tan(x)$

   $\Rightarrow du = \sec(x)\tan(x)\,dx$

3. Analyze in terms of u and du

   $\int \underbrace{(\sec(x))^2}_{u^2}\underbrace{\sec(x)\tan(x)dx}_{du} = \int u^2 du$

4. Integrate

   $\int u^2 du = \frac{u^3}{3} + C$

5. Re-express in terms of x

   i.e., $\int \sec^3(x)\tan(x)\,dx = \underbrace{\frac{(\sec(x))^3}{3} + C}_{\frac{u^3}{3}+C} = \frac{1}{3}\sec^3(x) + C$

13. Compute: $\int \frac{3t^2}{(2t^3+1)^{\frac{1}{2}}}dt \quad = \quad \int \left(2t^3+1\right)^{-\frac{1}{2}} 3t^2 dt$

↖ re-write ↗

(a) 1. U-sub appropriate?

1. Composite function? Yes!

$\left(2t^3+1\right)^{-\frac{1}{2}}$

Let $u = 2t^3 + 1$ (The "inner" function)

B. Approximate function/derivative pair? Yes!

$\underbrace{2t^3+1}_{\text{function}} \rightarrow \underbrace{3t^2}_{\text{deriv}}$

Let $u = 2t^3 + 1$ (The "function")

2. Compute du

$u = 2t^3 + 1$

$\frac{du}{dt} = 6t^2$

$du = 6t^2 dt$

$\frac{1}{2} du = 3t^2 dt$

3. Analyze in terms of u and du

$\int \underbrace{\left(2t^3+1\right)^{-\frac{1}{2}}}_{u^{-\frac{1}{2}}} \underbrace{3t^2 dt}_{\frac{1}{2}du} = \int u^{-\frac{1}{2}} \frac{1}{2} du = \frac{1}{2} \int u^{-\frac{1}{2}} du$

4. Integrate:

$= \frac{1}{2} \int u^{-\frac{1}{2}} du = \frac{1}{2}\left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right] + C = u^{\frac{1}{2}} + C$

5. Restate in terms of t

i.e., $\int \frac{3t^2}{(2t^3+1)^{\frac{1}{2}}}dt = \underbrace{\left(2t^3+1\right)^{\frac{1}{2}} + C}_{u^{\frac{1}{2}}+C}$

14. Compute:

$$\int \frac{\sin x}{\sqrt{\cos x}}dx \underset{\nwarrow \text{ re-write } \nearrow}{=} \int \frac{1}{(\cos(x))^{\frac{1}{2}}} \cdot \sin x\,dx \underset{\nwarrow \text{ re-write } \nearrow}{=} \int (\cos(x))^{-\frac{1}{2}} \cdot \sin x\,dx$$

(a) 1. U-sub appropriate?

1. Composite Function? Yes!

   $(\cos(x))^{-\frac{1}{2}}$

   Let $u = \cos(x)$ (The "inner" function)

2. Approx funct/deriv pair? Yes!

   $\underbrace{\cos(x)}_{\text{function}} \rightarrow \underbrace{\sin(x)}_{\text{deriv}}$

   Let $u = \cos(x)$ (The "function")

2. Compute du

$u = \cos(x)$

$\frac{du}{dx} = -\sin(x)$

$du = -\sin(x)\,dx$

$-du = \sin(x)\,dx$

3. Analyze in terms of u and du.

$$\int \underbrace{(\cos(x))^{-\frac{1}{2}}}_{u^{-\frac{1}{2}}} \underbrace{\sin(x)\,dx}_{-du} = \int u^{-\frac{1}{2}}\,(-du) = -\int u^{-\frac{1}{2}}\,du$$

4. Integrate

$$-\int u^{-\frac{1}{2}}\,du = -\left[\frac{u^{-\frac{1}{2}}}{\frac{1}{2}}\right] + C = -2u^{\frac{1}{2}} + C$$

5. Restate in terms of x

$$\text{i.e., } \int \frac{\sin x}{\sqrt{\cos x}}dx = \underbrace{-2(\cos(x))^{\frac{1}{2}} + C}_{-2u^{\frac{1}{2}}+C} = -2\sqrt{\cos(x)} + C$$

15. Given that $\ln(3) \approx 1.1$ and $\ln(4) \approx 1.4$, approximate the following:

(a) $\ln\left(\frac{3}{4}\right) =$

$\ln\left(\frac{3}{4}\right) = \ln(3) - \ln(4) \approx 1.1 - 1.4 = -0.3$

$\boxed{\ln\left(\frac{3}{4}\right) \approx -0.3}$

(b) $\ln(72) =$

$$\ln(72) = \ln\left(2 \cdot 4 \cdot 3^2\right) = \ln\left(4^{\frac{1}{2}} \cdot 4 \cdot 3^2\right)$$

$$= \ln\left(4^{\frac{1}{2}}\right) + \ln(4) + \ln\left(3^2\right) = \frac{1}{2}\ln(4) + \ln(4) + 2\ln(3)$$

$$\approx \frac{1}{2}(1.4) + 1.4 + 2(1.1) = 4.3$$

$\boxed{\ln(72) \approx 4.3}$

(c) $\ln(81) =$

$\ln(81) = \ln(34) = 4\ln(3) \approx 4(1.1) = 4.4$

$\boxed{\ln(81) \approx 4.4}$

16. Compute: $\int \frac{36x^2-28x}{6x^3-7x^2}dx \quad = \quad \int \frac{1}{6x^3-7x^2}\left(36x^2-28x\right)dx$

$\nwarrow$ re-write $\nearrow$

(a) 1. U-sub appropriate?

1. Composite Function? Yes!

$\frac{1}{6x^3-7x^2} = \left(6x^3-7x^2\right)^{-1}$

Let $u = 6x^3 - 7x^2$ (The "inner" function)

2. Approx funct/deriv pair? Yes!

$\underbrace{\left(6x^3-7x^2\right)}_{\text{function}} \rightarrow \underbrace{\left(36x^2-28x\right)}_{\text{deriv}}$

Let $u = 6x^3 - 7x^2$ (The "function")

2. Compute du

$$\begin{array}{|cccl|}\hline & u & = & \left(6x^3-7x^2\right) \\ \Rightarrow & \frac{du}{dx} & = & \left(18x^2-14x\right) \\ \Rightarrow & du & = & \left(18x^2-14x\right)dx \\ \Rightarrow & 2du & = & \left(36x^2-28x\right)dx \\ \hline\end{array}$$

3. Analyze in terms of u and du.

$\int \underbrace{\frac{1}{6x^3-7x^2}}_{\frac{1}{u}} \underbrace{\left(36x^2-28x\right)dx}_{2du} = \int \frac{1}{u} \cdot 2du = 2\int \frac{1}{u}du$

4. Integrate

$2\int \frac{1}{u}du = 2\ln|u| + C$

5. Restate in terms of x

$$\boxed{\text{i.e., } \int \frac{36x^2-28x}{6x^3-7x^2}dx = \underbrace{2\ln\left|6x^3-7x^2\right| + C}_{2\ln|u|+C}}$$

17. Compute:

$$\int \frac{e^{\cos(3x)}}{\csc(3x)} dx \underset{\nwarrow \ \text{re-write} \ \nearrow}{=} \int e^{\cos(3x)} \cdot \frac{1}{\csc(3x)}\, dx \underset{\nwarrow \ \text{re-write} \ \nearrow}{=} \int e^{\cos(3x)} \cdot \sin(3x)\; dx$$

(a) 1. U-sub appropriate?

1. Composite Function? Yes!

   $e^{\cos(3x)}$

   Let $u = \cos(3x)$ (The "inner" function)

2. Approx funct/deriv pair? Yes!

   $\underbrace{\cos(3x)}_{\text{function}} \rightarrow \underbrace{\sin(3x)}_{\text{deriv}}$

   Let $u = \cos(3x)$ (The "function")

2. Compute du

$$\begin{array}{|lll|} \hline & u & = & \cos(3x) \\ \Rightarrow & \frac{du}{dx} & = & -3\sin(3x) \\ \Rightarrow & du & = & -3\sin(3x)\,dx \\ \Rightarrow & -\frac{1}{3}du & = & \sin(3x)\,dx \\ \hline \end{array}$$

3. Analyze in terms of u and du.

$$\int e^{\cos(3x)} \cdot \frac{1}{\csc(3x)} dx = \int \underbrace{e^{\cos(3x)}}_{e^u} \cdot \underbrace{\sin(3x)\,dx}_{-\frac{1}{3}du} = \int e^u \cdot \left(-\frac{1}{3}du\right) = -\frac{1}{3}\int e^u du$$

4. Integrate

$$-\frac{1}{3}\int e^u du = -\frac{1}{3}e^u + C$$

5. Restate in terms of x

$$\boxed{\text{i.e., } \int \frac{e^{\cos(3x)}}{\csc(3x)} dx = \underbrace{-\frac{1}{3}e^{\cos(3x)} + C}_{\frac{1}{3}e^u + C}}$$

18. Compute: $\frac{d}{dx}\left[\ln\sqrt{\frac{3x^2+1}{2x^3+2}}\right] =$

$$\frac{d}{dx}\left[\ln\sqrt{\frac{3x^2+1}{2x^3+2}}\right] \underbrace{=}_{\text{rewrite}} \frac{d}{dx}\left[\ln\left(\frac{3x^2+1}{2x^3+2}\right)^{\frac{1}{2}}\right] \underbrace{=}_{\text{rewrite}} \frac{d}{dx}\left[\frac{1}{2}\ln\left(\frac{3x^2+1}{2x^3+2}\right)\right] \underbrace{=}_{\text{rewrite}} \frac{d}{dx}\left[\frac{1}{2}\left(\ln\left(3x^2+1\right)-\ln\left(2x^3+2\right)\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{3x^2+1}\cdot 6x - \frac{1}{2x^3+2}\cdot 6x^2\right] = \frac{3x}{3x^2+1} - \frac{3x^2}{2x^3+2}$$

$$\boxed{\text{i.e., } \frac{d}{dx}\left[\ln\sqrt{\frac{3x^2+1}{2x^3+2}}\right] = \frac{3x}{3x^2+1} - \frac{3x^2}{2x^3+2}}$$

19. Use the approximations $\ln 4 \approx 1.4$ and $\ln 9 \approx 2.2$ to approximate:

(a) $\ln\left(\frac{9}{4}\right) =$

$\ln\left(\frac{9}{4}\right) = \ln 9 - \ln 4 = 2.2 - 1.4 = 0.8$

$\boxed{\text{i.e., } \ln\left(\frac{9}{4}\right) \approx 0.8}$

(b) $\ln(81) =$

$\ln(81) = \ln\left(3^4\right) = 4\ln(3) \approx 4(1.1) = 4.4$

$\boxed{\ln(81) \approx 4.4}$

(c) $\ln 3 =$

$\ln 3 = \ln 9^{\frac{1}{2}} = \frac{1}{2}\ln 9 \approx \frac{1}{2}(2.2) = 1.1$

$\boxed{\ln 3 \approx 1.1}$

20. Compute: $\frac{d}{dx}\underbrace{\left[e^{\tan\left(3x^2\right)}\right]}_{e^u} = \underbrace{e^{\tan 3x^2}}_{e^u} \cdot \underbrace{\sec^2\left(3x^2\right)\cdot 6x}_{\frac{du}{dx}}$

$$\boxed{\text{i.e., } \frac{d}{dx}\left[e^{\tan\left(3x^2\right)}\right] = 6x\sec^2\left(3x^2\right)e^{\tan 3x^2}}$$

21. Compute: $\frac{d}{dx}\underbrace{\left[\ln\left(4x^5-3x^2\right)\right]}_{\ln u} = \underbrace{\frac{1}{4x^5-3x^2}}_{\frac{1}{u}} \cdot \underbrace{\left(20x^4-6x\right)}_{\frac{du}{dx}} = \frac{20x^4-6x}{4x^5-3x^2}$

$$\boxed{\text{i.e., } \frac{d}{dx}\left[\left[\ln\left(4x^5-3x^2\right)\right]\right] = \frac{1}{4x^5-3x^2}\cdot\left(20x^4-6x\right) = \frac{20x^4-6x}{4x^5-3x^2}}$$

22. Compute: $\int \frac{x^2+x}{4x^3+6x^2}dx \quad = \quad \int \frac{1}{4x^3+6x^2}\left(x^2+x\right)dx$

re-write

(a) 1. U-sub appropriate?

1. Composite Function? Yes!

$\frac{1}{4x^3+6x^2} = \left(4x^3+6x^2\right)^{-1}$

Let $u = 4x^3+6x^2$ (The "inner" function)

2. Approx funct/deriv pair? Yes!

$\underbrace{\left(4x^3+6x^2\right)}_{\text{function}} \rightarrow \underbrace{\left(x^2+x\right)}_{\text{deriv}}$

Let $u = 4x^3+6x^2$ (The "function")

2. Compute du:

$$\begin{array}{|lcl|} u & = & 4x^3+6x^2 \\ \frac{du}{dx} & = & \left(\left(12x^2+12x\right)\right) \\ du & = & \left(12x^2+12x\right)dx \\ \frac{1}{12}du & = & \left(x^2+x\right)dx \end{array}$$

3. Analyze in terms of u and du.

$$\int \underbrace{\frac{1}{4x^3+6x^2}}_{\frac{1}{u}} \underbrace{\left(x^2+x\right)}_{\frac{1}{12}du} dx = \int \frac{1}{u}\cdot\frac{1}{12}du = \frac{1}{12}\int\frac{1}{u}du$$

4. Integrate

$\frac{1}{12}\int\frac{1}{u}du = \frac{1}{12}\ln|u| + C$

5. Restate in terms of x

$$\boxed{\text{i.e., } \int \frac{x^2+x}{4x^3+6x^2}dx = \underbrace{\frac{1}{12}\ln\left|4x^3+6x^2\right| + C}_{\frac{1}{12}\ln|u|+C}}$$

23. Compute: $\int e^{\left(3x^3+6x^2\right)}\left(3x^2+4x\right)dx$

(a) 1. 1. Composite Function? Yes!

$e^{\left(3x^3+6x^2\right)}$

Let $u = \left(3x^3+6x^2\right)$ (The "inner" function)

2. Approx funct/deriv pair? Yes!

$\underbrace{\left(3x^3+6x^2\right)}_{\text{function}} \rightarrow \underbrace{\left(3x^2+4x\right)}_{\text{deriv}}$

Let $u = 3x^3+6x^2$ (The "function")

ii. Compute du:

$$\boxed{\begin{array}{ccl} u & = & 3x^3+6x^2 \\ \frac{du}{dx} & = & \left(9x^2+12x\right) \\ du & = & \left(9x^2+12x\right)dx \\ \frac{1}{3}du & = & \left(3x^2+4x\right)dx \end{array}}$$

iii. Analyze in terms of u and du.

$\int \underbrace{e^{\left(3x^3+6x^2\right)}}_{e^u}\underbrace{\left(3x^2+4x\right)dx}_{\frac{1}{3}du} = \int e^u \cdot \frac{1}{3}du = \frac{1}{3}\int e^u du$

iv. Integrate in terms of u

$\frac{1}{3}\int e^u du = \frac{1}{3}e^u + C$

v. Restate in terms of x

$$\boxed{\text{i.e., } \int e^{\left(3x^3+6x^2\right)}\left(3x^2+4x\right)dx = \underbrace{\frac{1}{3}e^{\left(3x^3+6x^2\right)} + C}_{\frac{1}{3}e^u+C}}$$

24. $\underbrace{\frac{d}{dx}\left[e^{\left(4x^2-3x\right)}\right]}_{\frac{d}{dx}[e^u]} = \underbrace{e^{\left(4x^2-3x\right)}}_{e^u} \cdot \underbrace{(8x-3)}_{\frac{du}{dx}}$

i.e., $\frac{d}{dx}\left[e^{\left(4x^2-3x\right)}\right] = (8x-3)\, e^{\left(4x^2-3x\right)}$

25. Compute: $\int \frac{1}{x^2+16} dx \quad = \quad \int \frac{1}{(4)^2+x^2} dx$

↖ re-write ↗

This closely fits a known form, namely: $\int \frac{1}{a^2+u^2} du = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$

Here:

	a^2	$=$	16
$\Rightarrow$	a	$=$	4
	u^2	$=$	x^2
$\Rightarrow$	u	$=$	x
$\Rightarrow$	$\frac{du}{dx}$	$=$	1
$\Rightarrow$	du	$=$	dx

Thus,

$\int \frac{1}{(4)^2+x^2} dx = \int \frac{1}{a^2+u^2} du = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C = \frac{1}{4} \tan^{-1}\left(\frac{x}{4}\right) + C$

i.e., $\int \frac{1}{x^2+16} dx = \frac{1}{4} \tan^{-1}\left(\frac{x}{4}\right) + C$

26. Compute: $\int \frac{e^{2x}}{1+e^{4x}} dx \quad = \quad \int \frac{1}{(1)^2+(e^{2x})^2} e^{2x} dx$

↖ re-write ↗

This closely fits the form $\int \frac{1}{a^2+u^2} du$.

Here:

	a^2	$=$	1
$\Rightarrow$	a	$=$	1
	u^2	$=$	$\left(e^{2x}\right)^2$
$\Rightarrow$	u	$=$	e^{2x}
$\Rightarrow$	$\frac{du}{dx}$	$=$	$2e^{2x}$
$\Rightarrow$	du	$=$	$2e^{2x} dx$
$\Rightarrow$	$\frac{1}{2} du$	$=$	$e^{2x} dx$

Thus, $\int \underbrace{\frac{1}{(1)^2+(e^{2x})^2}}_{\frac{1}{a^2+u^2}} \underbrace{e^{2x} dx}_{\frac{1}{2} du} = \int \frac{1}{a^2+u^2} \frac{1}{2} du = \frac{1}{2} \int \frac{1}{a^2+u^2} du = \frac{1}{2}\left(\frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right)\right) + C = \frac{1}{2} \tan^{-1}\left(e^{2x}\right) + C$

i.e., $\int \frac{e^{2x}}{1+e^{4x}} dx = \frac{1}{2} \tan^{-1}\left(e^{2x}\right) + C$

27. $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}dx \quad = \quad \int \frac{1}{\sqrt{1-(e^{2x})^2}} \cdot e^{2x}\,dx$

$\nwarrow$ re-write $\nearrow$

This closely fits a known form, namely: $\int \frac{1}{\sqrt{a^2-u^2}}du = \arcsin\left(\frac{u}{a}\right) + C$

Here:

$$\begin{array}{|lrcl|} & a^2 & = & 1 \\ \Rightarrow & a & = & 1 \\ & u^2 & = & e^{4x} \\ \Rightarrow & u & = & e^{2x} \\ \Rightarrow & \frac{du}{dx} & = & 2e^{2x} \\ \Rightarrow & du & = & 2e^{2x}dx \\ \Rightarrow & \frac{1}{2}du & = & e^{2x}dx \end{array}$$

Thus, $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}dx = \int \underbrace{\frac{1}{\sqrt{1-(e^{2x})^2}}}_{\frac{1}{\sqrt{a^2-u^2}}} \underbrace{e^{2x}dx}_{\frac{1}{2}du} = \frac{1}{2}\int \frac{1}{\sqrt{a^2-u^2}}du = \frac{1}{2}\sin^{-1}\left(\frac{u}{a}\right) + C = \frac{1}{2}\sin^{-1}\left(e^{2x}\right) + C$

> i.e., $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}dx = \frac{1}{2}\sin^{-1}\left(e^{2x}\right) + C$

28. $\frac{d}{dx}\left[\underbrace{\cos^{-1}(3x)}_{\cos^{-1}(u)}\right] = \underbrace{-\frac{1}{\sqrt{1-(3x)^2}}}_{-\frac{1}{\sqrt{1-u^2}}} \cdot \underbrace{3}_{\frac{du}{dx}} = -\frac{3}{\sqrt{1-9x^2}}$

> i.e., $\frac{d}{dx}\left[\cos^{-1}(3x)\right] = -\frac{3}{\sqrt{1-9x^2}}$

29. $\int \frac{1}{\sqrt{e^{2x}-5}}dx =$ We will try to make this fit the form: $\int \frac{1}{u\sqrt{\underbrace{u^2}_{\text{function of } x} - \underbrace{a^2}_{\text{constant}}}}du = \frac{1}{a}\text{ arcsec}\left(\frac{|u|}{a}\right) + C$

If this analysis is correct, then:

a^2	$=$	5
a	$=$	$\sqrt{5}$
u^2	$=$	$e^{2x} = (e^x)^2$
u	$=$	e^x
$\frac{du}{dx}$	$=$	e^x
du	$=$	$e^x dx$

We have two apparent problems:

[1] our integrand does not have a "u" in front of the radical

[2] our integrand does not have a "du," where $du = e^x dx$

We will attempt to fix these problems, one at a time.

First, we will put a "u" in front of the radical.

$\int \frac{1}{\sqrt{e^{2x}-5}}dx \Rightarrow \int \frac{1}{e^x\sqrt{e^{2x}-5}}dx$

This means that we have divided the integrand by e^x.

To atone for this mathematical indescretion, we must **multiply** the integrand by e^x.

This yields:

$\int \frac{1}{\sqrt{e^{2x}-5}}dx \Rightarrow \int \frac{1}{e^x\sqrt{e^{2x}-5}}dx \Rightarrow \int \frac{1}{e^x\sqrt{e^{2x}-5}}e^x dx$

Thus we have:

$\int \frac{1}{\sqrt{e^{2x}-5}}dx = \int \frac{1}{e^x\sqrt{e^{2x}-5}}e^x dx$

Notice that by fixing the first problem, we have also fixed the second problem. So NOW we can integrate.

$$\int \underbrace{\frac{1}{e^x\sqrt{e^{2x}-5}}}_{\frac{1}{u\sqrt{u^2-a^2}}}\underbrace{e^x dx}_{du} = \int \frac{1}{u\sqrt{u^2-a^2}}du = \frac{1}{a}\sec^{-1}\left(\frac{|u|}{a}\right) + C = \frac{1}{\sqrt{5}}\sec^{-1}\left(\frac{|e^x|}{\sqrt{5}}\right) + C = \frac{1}{\sqrt{5}}\sec^{-1}\left(\frac{e^x}{\sqrt{5}}\right) + C$$

> i.e., $\int \frac{1}{\sqrt{e^{2x}-5}}dx = \frac{1}{\sqrt{5}}\sec^{-1}\left(\frac{e^x}{\sqrt{5}}\right) + C$

30. Compute: $\frac{d}{dx}\left[\text{arccot}\left(3x^2\right)\right] = \underbrace{\frac{d}{dx}\left[\text{arccot}\left(3x^2\right)\right]}_{\frac{d}{dx}[\text{arccot}(u)]} = -\underbrace{\frac{1}{1+\left(3x^2\right)^2}}_{\frac{1}{1+u^2}} \cdot \underbrace{6x}_{\frac{du}{dx}} = -\frac{6x}{1+9x^4}$

> i.e., $\frac{d}{dx}\left[\text{arccot}\left(3x^2\right)\right] = -\frac{6x}{1+9x^4}$

31. Write the given equation in algebraic form.

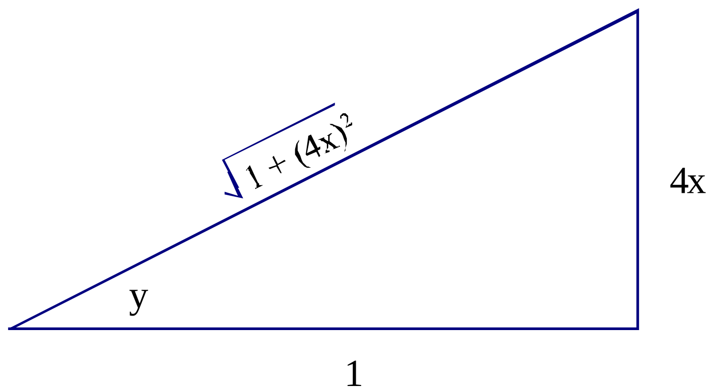
$z = \sec(\arctan(4x))$

Let $y = \arctan(4x)$

This is the same as saying "y is the angle whose tangent is $4x$."

i.e., $\tan(y) = 4x$.

Let's draw a triangle to depict this relationship.



By Pythagorean's Theorem, $(opp)^2 + (adj)^2 = (hyp)^2$

$\Rightarrow (hyp) = \sqrt{(opp)^2 + (adj)^2} = \sqrt{1^2 + (4x)^2}$.

Recall: we want $z = \sec(\arctan(4x)) = \sec(y)$

From the picture, $z = \sec(y) = \frac{hyp}{adj} = \frac{\sqrt{1^2+(4x)^2}}{1} = \sqrt{1^2 + (4x)^2}$

Hence, $z = \sec(\arctan(4x)) = \sqrt{1^2 + (4x)^2} = \sqrt{1 + 16x^2}$

$\boxed{\text{i.e., } z = \sqrt{1 + (4x)^2} = \sqrt{1 + 16x^2}}$