MTH 3318 - Test #3

FALL 2022

Pat Rossi Name ______________________________

Instructions. Show your work completely. Document your work well.

Remark 1 *For problems 1 - 3, prove two.*

1. Prove or disprove: $f : \Re \longrightarrow \Re$ given by $f(x) = 3x^2 - 2$, is one to one.

2. Prove or disprove: $f : \Re \longrightarrow \Re$ given by $f(x) = 5x + 1$, is one to one.

3. Prove or disprove: $f : \Re \longrightarrow \Re$ given by $f(x) = x^3 + 3$, is one to one.

Remark 2 *For problems 4 - 6, prove two.*

4. Prove or disprove: $f : \Re \longrightarrow \Re$ given by $f(x) = 3x^2 - 2$, is onto.

5. Prove or disprove: $f : \Re \longrightarrow \Re$ given by $f(x) = 5x + 1$, is onto.

6. Prove or disprove: $f : \Re \longrightarrow \Re$ given by $f(x) = x^3 + 3$, is onto.

Remark 3 *For problems 7-8.*

7. **Prove:** The set of odd natural numbers, $\mathbf{O} = \{1, 3, 5, 7, \ldots, 2n-1, \ldots\}$, is countably infinite (i.e., denumerable).

8. **Prove:** The set of Integers, $\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, \ldots\}$, is countably infinite (i.e., denumerable).

Remark 4 *For problems 9 - 10, prove either one.*

9. The set of positive rational numbers $\mathbb{Q}^+$ is countably infinite (i.e., denumerable).

10. The set of real numbers in the interval $[0, 1]$ is uncountable (i.e., non-denumerable).

Remark 5 ***Select TWO problems from problems 11 - 15.***

11. Prove or disprove: $x \in \mathbf{Q}$ and $y \in \mathbf{Q} \Rightarrow x + y \in \mathbf{Q}$

12. Prove or disprove: $x \in \mathbf{Q}$ and $y \in \mathbf{Q}^c \Rightarrow x + y \in \mathbf{Q}^c$

13. Prove or disprove: $x \in \mathbf{Q}$ and $y \in \mathbf{Q} \Rightarrow xy \in \mathbf{Q}$

14. Prove or disprove: $x \in \mathbf{Q}$ and $y \in \mathbf{Q}^c \Rightarrow x + y \in \mathbf{Q}^c$

15. Prove or disprove: $x \in \mathbf{Q}^c$ and $y \in \mathbf{Q}^c \Rightarrow xy \in \mathbf{Q}^c$