MTH 6610 - History of Math Reading Assignment #6 - Answers

Term V - 2022

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Instructions. Read pages 183 - 208 to find the answers to these questions in your reading.

1. In what fields of study did Eratosthenes excel?

   Geography, Philosophy, History, Astronomy, Mathematics, Literary Criticism, and Poetry.

2. What was noteworthy about his three volume work, *Geographica? (*Name at least three things*)*

   i. It was the first scientific attempt to put geographical studies on a sound mathematical foundation.

   ii. He discussed the arguments that implied that the earth was spherical

   iii. He described the positions (given the assumption of a spherical earth) of various land masses of the know world.

   iv. He was the first to use a grid of longitudinal meridians and parallels of latitude.

3. What tool for use in Number Theory may be considered Eratosthenes' most significant mathematical contribution?

   The "sieve of Erathosthenes." This is constructed by listing integers from 2 to n, and then removing all multiples of each prime (except for the prime itself!).

4. For what Mathematical/Geographical contribution is Eratosthenes best remembered today?

   Devising a practical and simple method for calculating the earth's circumference.

Remark: Eratosthenes was not considered, by his contemporaries, to be a "top notch scholar/mathematician." And yet he made a sizable and noteworthy contribution to mathematics, geography, and civilization. The point here is that our students don't have to be "the best" at what they do, in order to "belong in their chosen field," in order to be successful, and in order to "find their niche in life" - they just have to do their best and be reasonably good. Eratosthenes was not considered to be among "the best" mathematicians of his era, but his mathematical genius was certainly augmented by his broad background, his expertise in these areas, and also by his inspired creativity (of which he apparently had an abundance.)

Remark: A second point, related to the first, is that our students benefit by being exposed to, and educated in, a number of different areas. Consider Eratosthenes as an example. The notion of a spherical earth may have first been brought to his attention during his studies as a *historian.* Certainly his background in *Astronomy* made the idea of a spherical earth plausible and may have even made the idea a certainty in his mind. Being a *mathematician,* he was able to devise a simple, practical experiment and calculate, using his experimental data, the circumference of the earth. His background as both a *mathematician and geographer* certainly allowed him to come up with the notion of a geographical grid on which to plot land masses, etc., whereas, those who were not experts in *both* disciplines, would not have been able to devise this clever scheme. In fact, even *AFTER* Eratosthenes devised this longitudinal/latitudinal grid, it took mathematicians more than 1800 years to come up with the idea of a coordinate plane (i.e. x-y plane).

Remark: The third and final point, related to the first two, is that what genius Eratosthenes had was augmented by his creativity. Some people who are genuinely brilliant and extremely quick to understand even the most complicated topics, may not possess much in the way of creativity. Although they are genuinely brilliant, they may not produce as much as one would expect such a brilliant person to produce. On the other hand, some people of "modest genius," are also quite creative and inspired. Although not as quick as their peers to fully grasp a deep topic, they seem to be able to think of things and come up with ideas that no one else would ever think of. Their genius is augmented by their inspired creativity. Standardized tests can never reveal such inspired creativity. Our students may not score in the top percentiles of standardized tests, and yet may have quite a future ahead of them because of their creativity. So if our students are good, but not brilliant, there is still a niche for them in Mathematics, as long as they do their best.

5. What is noteworthy of Aristarchus of Samos?

   Despite the prevailing cultural opinion that the earth was the center of the universe, he proposed the "heliocentric hypothesis," namely that the planets revolved about the sun.

6. What was noteworthy of Claudius Ptolemy's maps?

   He sought to reproduce the surface of the earth (a *spherical* surface) on a flat, two-dimensional surface by representing the parallels and meridians as curved lines, with the meridians converging to the poles.

7. The inaccuracy of Claudius Ptolemy's maps gave impetus to what achievement, centuries later?

   Ha! This is a great one! Ptolemy's diminution of the distance between Europe and Asia by some 50^{o} latitude fortified Columbus' belief that he could easily reach the

Orient by sailing west across the Atlantic - prompting the "discovery" of the "New World."

8. What details are known about Archimedes' life?

   Lived c. 287-212 BC. He was the son of astronomer Phidias and was born in Syracuse, a Greek settlement on the SE coast of Sicily. (At that time the largest city in the Hellenistic world.) Almost certainly he visited the Alexandrian Museum (he was in correspondence with Alexandrian scholars) and he very likely that he studied there. He spent most of his life in Syracuse under the mentorship of Hieron. He is known for directing the defense of Syracuse during the Second Punic War, during which, he was killed by a Roman Soldier - even though Roman soldiers were given explicit orders that Archimedes not be killed.

9. What discoveries regarding spheres and cylinders are attributed to Archimedes? (We're looking for two "discoveries.")

   i. The surface area of a sphere is four times the area of a great circle of the sphere (i.e., $A = 4\pi r^2$).

   ii. Given a sphere of diameter D, a right circular cylinder of height D, inscribed about the sphere, has a volume three halves the volume of the sphere. (i.e., $V = \frac{3}{2} \cdot \frac{4}{3}\pi r^3 = 2\pi r^3$).

10. Briefly (but not too briefly describe how Archimedes was able to approximate the value of pi to a value of $\frac{22}{7}$.

    He used the fact that the circumference of a circle lies between the perimeters of inscribed and circumscribed regular polygons of n sides. Archimedes successively inscribed and circumscribed regular polygons of 6, 12, 24, 48, and 96 sides within and without the circle. (The hexagon was most easily inscribed of all n-gons. The he just doubled the number of sides each time.)

    Letting p_n denote the circumference of an inscribed n-gon, and letting P_n denote the circumference of a circumscribed n-gon, he had:

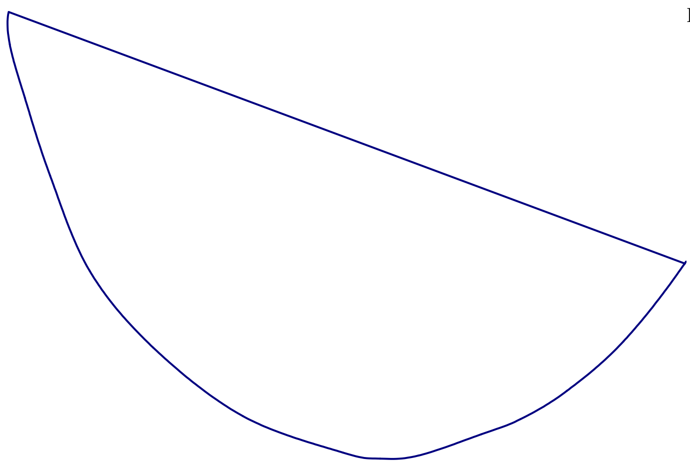
    $$p_6 < p_{12} < p_{24} < p_{48} < p_{96} < \ldots < p_n < C < P_n < \ldots < P_{96} < P_{48} < P_{24} < P_{12} < P_6$$

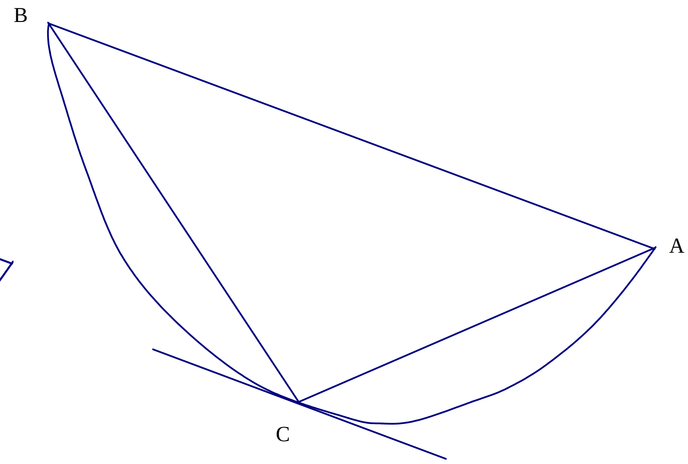
    Computing p_{96} and P_{96}, he had:

    $$3 + \frac{10}{71} < \pi < 3 + \frac{1}{7}$$

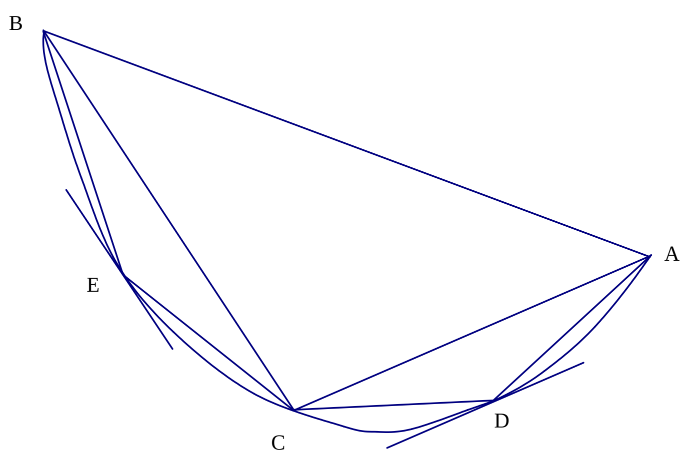
11. Describe how Archimedes was able to calculate the area bounded by the arc of a parabola, and the chord connecting the endpoints of the arc of the parabola.

    Given the region bounded by a parabolic arc and a chord connecting the endpoints of the arc (see below left), Archimedes inscribed a triangle whose base was the chord and whose remaining vertex was a point on the arc. This remaining vertex was chosen to be the point of intersection of the arc and the tangent line to the arc that happened to be parallel to chord (see below right).





    The area of the triangle was computed, and then two more triangles were inscribed within the region, in such a way that each side of the original triangle served as the base of the two new triangles. The remaining vertex of each triangle was chosen to be the point of intersection of the arc and the tangent line to the arc that happened to be parallel to base of the triangle (see below).



    This process was repeated over and over again, hypothetically "exhausting" all of the bounded area.

It turns out that the second set of triangles had a combined area equal to $\frac{1}{4}$ the area of the original triangle and that the third set of triangles had a combined area equal to $\frac{1}{4}$ the combined area of the second set of triangles, etc. In turn, each set of triangles has a combined area equal to $\frac{1}{4}$ the combined area of the preceding set of triangles. Thus, if we let Δ equal the area of the original triangle, then the sum of the areas of the inscribed triangles is equal to:

$$\Delta + \frac{1}{4}\Delta + \frac{1}{4}\left(\frac{1}{4}\Delta\right) + \ldots = \Delta + \frac{1}{4}\Delta + \left(\frac{1}{4}\right)^2 \Delta + \ldots$$

Archimedes took this finite geometric series to a sufficient number of terms to guarantee the desired accuracy.

$A \approx \frac{4}{3}\Delta$

12. Archimedes' use of the "Method of Exhaustion" is very similar to later mathematical innovations. What are they?

    The limit and Calculus

13. The lack of what "tool" made it impossible for Archimedes to progress beyond the results that he achieved using the Method of Exhaustion?

    Calculus (specifically, Integral Calculus)