MTH 4441 - HW 5 - Isomorphisms

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In Exercises 1-12, determine whether the two groups are isomorphic. If they aren't, give at least one reason why. If they are, justify your answer either by exhibiting an isomorphism between the two groups, or by proving that they are isomorphic by some other method.

1. $(\mathbb{Z}, +)$ and $(2\mathbb{Z}, +)$
2. $(2\mathbb{Z}, +)$ and $(6\mathbb{Z}, +)$
3. $(\mathbb{Z}_2 \times \mathbb{Z}_2, \oplus)$ and the group $(G, *)$ whose group table is given below:

$*$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

4. $(\mathbb{Z}_2 \times \mathbb{Z}_2, \oplus)$ and the group $(G, *)$ whose group table is given below:

$*$	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

5. The groups $(G, *)$ and $(H, *)$, whose group tables are given below:

$*$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

$*$	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

6. The groups $(G, *)$ and $(H, *)$, whose group tables are given below:

$*$	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

$*$	e	a	b	c
e	e	a	b	c
a	a	c	e	b
b	b	e	c	a
c	c	b	a	e

7. The groups $(G, *)$ and $(H, *)$, whose group tables are given below:

$*$	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

$*$	e	a	b	c	d
e	e	a	b	c	d
a	a	b	c	d	e
b	b	c	d	e	a
c	c	d	e	a	b
d	d	e	a	b	e

8. The groups $(\mathbb{Z}_4, \oplus)$ and $(G, *)$, whose group table is given below:

$*$	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

9. The groups $(\mathbb{Z}_6, \oplus)$ and $(H, *)$, whose group tables are given below:

$\oplus$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

$*$	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	b	e	d	f	c
b	b	e	a	f	c	d
c	c	f	d	e	b	a
d	d	c	f	a	e	b
f	f	d	c	b	a	e

10. The groups $(\mathbb{Z}_8, \oplus)$ and $(H, *)$, whose group tables are given below:

$\oplus$	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

$*$	1	-1	i	$-i$	j	$-j$	k	$-k$
1	1	-1	i	$-i$	j	$-j$	k	$-k$
-1	-1	1	$-i$	i	$-j$	j	$-k$	k
i	i	$-i$	1	-1	k	$-k$	$-j$	j
$-i$	$-i$	i	-1	1	$-k$	k	j	$-j$
j	j	$-j$	$-k$	k	-1	1	i	$-i$
$-j$	$-j$	j	k	$-k$	1	-1	$-i$	i
k	k	$-k$	j	$-j$	$-i$	$-i$	-1	1
$-k$	$-k$	k	$-j$	j	i	i	1	-1

11. $(\mathbb{Z}_2 \times \mathbb{Z}_3, \oplus)$ and $(\mathbb{Z}_3 \times \mathbb{Z}_2, \oplus)$

12. $(\mathbb{Z}_3 \times \mathbb{Z}_2, \oplus)$ and $(\mathbb{Z}_6, \oplus)$

In Exercises 13-17, determine whether the given function is an isomorphism between the two groups. If it is, show that it satisfies all of the properties of an isomorphism. If it is not, give at least one reason why the function is NOT an isomorphism.

13. $f : (\mathbb{R} \setminus \{0\}, \cdot) \to (\mathbb{R}^+, \cdot)$, given by $f(x) = |x|$

14. $f : (\mathbb{R} \setminus \{0\}, \cdot) \to (\mathbb{Z}_2 \times \mathbb{R}^+, \cdot)$, given by $f(x) = \begin{cases} (x, 0) & \text{if } x > 0 \\ (-x, 1) & \text{if } x < 0 \end{cases}$,

    where, $*$ in $(\mathbb{Z}_2 \times \mathbb{R}^+, *)$ is addition modulo 2 in the first coordinate, and multiplication in the second coordinate.

15. $f : (\mathbb{R}^+, \cdot) \to (\mathbb{R}^+, \cdot)$, given by $f(x) = \sqrt{x}$

16. $f : (\mathbb{Z}_2 \times \mathbb{Z}_3, \oplus)$ and $(\mathbb{Z}_3 \times \mathbb{Z}_2, \oplus)$ given by $f((a, b)) = (b, a)$

17. $f : (\mathbb{R}^+, \cdot) \to (\mathbb{R}, +)$, given by $f(x) = \ln(x)$