MTH 1125 - Test 2

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Pat Rossi   Name ______________________

Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\frac{d}{dx}\left[2x^6+3x^4+4x^3+6x^2+12x+24\sqrt{x}+48\right]$

$= \frac{d}{dx}\left[2x^6+3x^4+4x^3+6x^2+12x+24x^{\frac{1}{2}}+48\right]$

$= 2\left[6x^5\right]+3\left[4x^3\right]+12\left[3x^2\right]+6\left[2x\right]+12+24\left[\frac{1}{2}x^{-\frac{1}{2}}\right]$

$= 12x^5+12x^3+12x^2+12x+12+12x^{-\frac{1}{2}}$

i.e., $\frac{d}{dx}\left[2x^6+3x^4+4x^3+6x^2+12x+24\sqrt{x}+48\right]=12x^5+12x^3+12x^2+12x+12+12x^{-\frac{1}{2}}$

2. Compute: $\frac{d}{dx}\left[\left(8x^5+16x\right)\sec(x)\right]=$

$$\frac{d}{dx}\left[\underbrace{\left(8x^5+16x\right)}_{1^{st}}\underbrace{\sec(x)}_{2^{nd}}\right]=\underbrace{\left(40x^4+16\right)}_{1^{st}\text{ prime}}\cdot\underbrace{\sec(x)}_{2^{nd}}+\underbrace{\left(\sec(x)\tan(x)\right)}_{2^{nd}\text{ prime}}\cdot\underbrace{\left(8x^5+16x\right)}_{1^{st}}$$

$\frac{d}{dx}\left[\left(8x^5+16x\right)\sec(x)\right]=\left(40x^4+16\right)\sec(x)+\sec(x)\tan(x)\left(8x^5+16x\right)$

3. Compute: $\frac{d}{dx}\left[\frac{\tan(x)}{25x^4+33x^3+98x}\right]=$

$$\frac{d}{dx}\left[\frac{\overbrace{\tan(x)}^{\text{top}}}{\underbrace{25x^4+33x^3+98x}_{\text{Bottom}}}\right]=\frac{\overbrace{\sec^2(x)}^{\text{top prime}}\cdot\overbrace{\left(25x^4+33x^3+98x\right)}^{\text{bottom}}-\overbrace{\left(100x^3+99x^2+98\right)}^{\text{bottom prime}}\cdot\overbrace{\tan(x)}^{\text{top}}}{\underbrace{\left(25x^4+33x^3+98x\right)^2}_{\text{bottom squared}}}$$

i.e., $\frac{d}{dx}\left[\frac{\tan(x)}{25x^4+33x^3+98x}\right]=\frac{\sec^2(x)\left(25x^4+33x^3+98x\right)-\left(100x^3+99x^2+98\right)\tan(x)}{\left(25x^4+33x^3+98x\right)^2}$

4. Compute: $\frac{d}{dx}\left[\left(10x^5+\cos\left(x\right)\right)^{10}\right] =$ This is the derivative of a function raised to a power.

$$\frac{d}{dx}\left[\left(10x^5+\cos\left(x\right)\right)^{10}\right] = \underbrace{10\left(10x^5+\cos\left(x\right)\right)^9}_{\text{power rule as usual}} \cdot \underbrace{\left(50x^4-\sin\left(x\right)\right)}_{\text{derivative of inner}}$$

i.e., $\frac{d}{dx}\left[\left(10x^5+\cos\left(x\right)\right)^{10}\right] = 10\left(10x^5+\cos\left(x\right)\right)^9\left(50x^4-\sin\left(x\right)\right)$

5. Given that $f\left(x\right)=4x^2-2x+2$, give the *equation* of the line tangent to the graph of $f\left(x\right)$ at the point $\left(1,4\right)$.

We need two things:

   i. A point on the line (We have that: $\left(x_1,y_1\right)=\left(1,4\right)$)

   ii. The slope of the line (This is $f'\left(x_1\right)$)

   $f'\left(x\right)=8x-2$

At the point $\left(x_1,y_1\right)=\left(1,4\right)$, **the slope is** $f'\left(1\right)=8\left(1\right)-2=6$

We will use the Point-Slope equation of a line:

$y-y_1=m\left(x-x_1\right)$ (Where m is the slope and $\left(x_1,y_1\right)$ is a known point on the line.)

Thus, the equation of the line tangent to the graph of $f\left(x\right)$ is:

$y-4=6\left(x-1\right)$

The equation of the line tangent is $y-4=6\left(x-1\right)$

6. Given that $x = 8t^2 + 8t$ and that $t = \sec(y)$; compute $\frac{dx}{dy}$ **using the Liebniz form of the Chain Rule.** (In particular, when doing this exercise, *write the Liebniz form of the chain rule, that you are going to use, explicitly.)*

   We know:

   $\frac{dx}{dt} = 16t + 8$

   $\frac{dt}{dy} = \sec(y)\tan(y)$

   We want: $\frac{dx}{dy}$

   By the Liebniz form of the Chain Rule:

$$\frac{dx}{dy} = \frac{dx}{dt}\frac{dt}{dy} = (16t+8)\sec(y)\tan(y) = \underbrace{(16t+8)\sec(y)\tan(y)}_{\substack{\text{express solely in terms of}\\ \text{independent variable } y}} = (16\sec(y)+8)\sec(y)\tan(y)$$

i.e. $\frac{dx}{dy} = (16\sec(y)+8)\sec(y)\tan(y)$

7. Compute: $\frac{d}{dx}\left[\sec\left(4x^2+8x+6\right)\right] =$

Outer:	=	$\sec(\quad)$
Deriv. of outer	=	$\sec(\quad)\tan(\quad)$

$$\frac{d}{dx}\left[\begin{array}{c} \sec\left(\underbrace{4x^2+8x+6}\right) \\ \uparrow \qquad\qquad \uparrow \\ \text{outer} \quad \text{inner} \end{array}\right] = \underbrace{\sec\left(4x^2+8x+6\right)\tan\left(4x^2+8x+6\right)}_{\substack{\text{Deriv of outer,}\\ \text{eval. at inner}}} \cdot \underbrace{(8x+8)}_{\substack{\text{deriv. of}\\ \text{inner}}}$$

i.e., $\frac{d}{dx}\left[\sec\left(4x^2+8x+6\right)\right] = \sec\left(4x^2+8x+6\right)\tan\left(4x^2+8x+6\right)(8x+8)$

8. Compute: $\frac{d}{dx}\left[\left(\frac{2x^5+6}{5x^2+10x}\right)^{10}\right] =$ In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE.

$$\frac{d}{dx}\left[\underbrace{\left(\frac{2x^5+6}{5x^2+10x}\right)^{10}}_{(g(x))^n}\right] = \underbrace{10\left(\frac{2x^5+6}{5x^2+10x}\right)^{9}}_{\substack{\text{power rule} \\ \text{as usual}}} \cdot \underbrace{\left(\frac{d}{dx}\left[\frac{2x^5+6}{5x^2+10x}\right]\right)}_{\substack{\text{deriv of} \\ \text{inner Function}}}$$

$$= 10\left(\frac{2x^5+6}{5x^2+10x}\right)^{9} \underbrace{\frac{(10x^4)(5x^2+10x) - (10x+10)(2x^5+6)}{(5x^2+10x)^2}}_{\substack{\text{quotient} \\ \text{rule}}}$$

i.e., $\frac{d}{dx}\left[\left(\frac{2x^5+6}{5x^2+10x}\right)^{10}\right] = 10\left(\frac{2x^5+6}{5x^2+10x}\right)^{9} \cdot \frac{(10x^4)(5x^2+10x) - (10x+10)(2x^5+6)}{(5x^2+10x)^2}$

9. Compute: $\frac{d}{dx}\left[\tan^6(9x^2+18x)\right] =$ Re-write!

$\frac{d}{dx}\left[(\tan(9x^2+18x))^6\right]$ This is the derivative of a function, raised to a power

$$\frac{d}{dx}\left[(\tan(9x^2+18x))^6\right] = \underbrace{6\left(\tan\left(9x^2+18x\right)\right)^5}_{\substack{\text{power rule} \\ \text{as usual}}} \cdot \underbrace{\left(\frac{d}{dx}\left[\tan\left(9x^2+18x\right)\right]\right)}_{\substack{\text{derivative} \\ \text{of inner}}}$$

$$= 6\left(\tan(9x^2+18x)\right)^5 \cdot \underbrace{\left(\sec^2\left(9x^2+18x\right)\right) \cdot (18x+18)}_{\substack{\text{Chain} \\ \text{Rule}}}$$

i.e., $\frac{d}{dx}\left[\tan^6(9x^2+18x)\right] = 6\left(\tan(9x^2+18x)\right)^5 \cdot \left(\sec^2(9x^2+18x)\right) \cdot (18x+18)$

10. Given that $T'(x) = \frac{1}{1+x^2}$; compute $\frac{d}{dx}\left[T(\tan(x))\right]$

Outer:	$=$	$T(\quad)$
Deriv. of outer	$=$	$\frac{1}{1+(\quad)^2}$

$$\frac{d}{dx}\Bigg[\underset{\substack{\uparrow \\ \text{outer}}}{T}\Big(\underset{\substack{\uparrow \\ \text{inner}}}{\underbrace{\tan(x)}}\Big)\Bigg] = \underbrace{\frac{1}{1+(\tan(x))^2}}_{\substack{\text{Deriv of outer,} \\ \text{eval. at inner}}} \cdot \underbrace{\sec^2(x)}_{\substack{\text{deriv. of} \\ \text{inner}}} = \frac{\sec^2(x)}{1+(\tan(x))^2}$$

i.e., $\frac{d}{dx}\left[T(\tan(x))\right] = \frac{\sec^2(x)}{1+(\tan(x))^2}$

11. Given that $f(x) = x^2 - 4x + 2$, compute $f'(x)$ **using the definition of derivative.** (i.e., using the "limit process.")

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[(x+\Delta x)^2 - 4(x+\Delta x) + 2\right] - \left[x^2 - 4x + 2\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[\left(x^2 + 2x\Delta x + \Delta x^2\right) - 4(x+\Delta x) + 2\right] - \left[x^2 - 4x + 2\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[x^2 + 2x\Delta x + \Delta x^2 - 4x - 4\Delta x + 2\right] - \left[x^2 - 4x + 2\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2 - 4\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x - 4)}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x - 4) = 2x + (0) - 4 = 2x - 4$$

i.e., $f'(x) = 2x - 4$