MTH 1125 2pm Class - Test #4 - Solutions

Fall 2023

Pat Rossi Name ______________________________

Show CLEARLY how you arrive at your answers!

1. **Compute:** $\int \left(40x^3 + 24x^2 - 12x + 4 + 3\sqrt{x}\right) dx = 4x - 6x^2 + 8x^3 + 10x^4 + 2x^{\frac{3}{2}} =$

$$\int \left(40x^3 + 24x^2 - 12x + 4 + 3\sqrt{x}\right) dx = \int \left(40x^3 + 24x^2 - 12x + 4 + 3x^{\frac{1}{2}}\right) dx$$

$$= 40\left[\frac{x^4}{4}\right] + 24\left[\frac{x^3}{3}\right] - 12\left[\frac{x^2}{2}\right] + 4x + 3\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right] + C = 10x^4 + 8x^3 - 6x^2 + 4x + 2x^{\frac{3}{2}} + C$$

i.e., $\int \left(40x^3 + 24x^2 - 12x + 4 + 3\sqrt{x}\right) dx = 10x^4 + 8x^3 - 6x^2 + 4x + 2x^{\frac{3}{2}} + C$

2. **Compute:** $\int \left(6x^2+18x+2\right)^5 (2x+3)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(6x^2+18x+2\right)^5$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = (6x^2+18x+2)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{(6x^2+18x+2)}_{\text{function}} - - - - \rightarrow \underbrace{(2x+3)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = (6x^2+18x+2)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 6x^2+18x+2 \\ \Rightarrow \frac{du}{dx} &= 12x+18 \\ \Rightarrow du &= (12x+18)\,dx \\ \Rightarrow \frac{1}{6}du &= (2x+3)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\left(6x^2+18x+2\right)^5}_{u^5}\underbrace{(2x+3)\,dx}_{\frac{1}{6}du} = \int u^5 \tfrac{1}{6}du = \tfrac{1}{6}\int u^5 du$$

4. Integrate (in terms of u).

$$\tfrac{1}{6}\int u^5 du = \tfrac{1}{6}\left[\tfrac{u^6}{6}\right] + C = \tfrac{1}{36}u^6 + C$$

5. Re-express in terms of the original variable, x.

$$\int \left(6x^2+18x+2\right)^5 (2x+3)\,dx = \underbrace{\frac{1}{36}\left(6x^2+18x+2\right)^6 + C}_{\frac{1}{36}u^6+C}$$

i.e., $\int \left(6x^2+18x+2\right)^5 (2x+3)\,dx = \frac{1}{36}\left(6x^2+18x+2\right)^6 + C$

3. **Compute:** $\int \left(8\sin(x) - 2\sec^2(x) + 5\csc(x)\cot(x)\right) dx =$

$\int \left(8\sin(x) - 2\sec^2(x) + 5\csc(x)\cot(x)\right) dx = 8\left[-\cos(x)\right] - 2\left[\tan(x)\right] + 5\left[-\csc(x)\right] + C$

$= -8\cos(x) - 2\tan(x) - 5\csc(x) + C$

i.e., $\int \left(8\sin(x) - 2\sec^2(x) + 5\csc(x)\cot(x)\right) dx = -8\cos(x) - 2\tan(x) - 5\csc(x) + C$

4. **Compute:** $\int \cos\left(9x^2+15x+2\right)(6x+5)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\cos\left(9x^2+15x+2\right)$

   $\nearrow$ outer $\quad\nwarrow$ inner

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(9x^2+15x+2\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(9x^2+15x+2\right)}_{\text{function}} - - - - \rightarrow \underbrace{(6x+5)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(9x^2+15x+2\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 9x^2+15x+2 \\ \Rightarrow \frac{du}{dx} &= 18x+15 \\ \Rightarrow du &= (18x+15)\,dx \\ \Rightarrow \tfrac{1}{3}du &= (6x+5)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\cos\left(9x^2+15x+2\right)}_{\cos(u)}\underbrace{(6x+5)\,dx}_{\frac{1}{3}du} = \int \cos(u)\ \tfrac{1}{3}du = \tfrac{1}{3}\int \cos(u)\,du$$

4. Integrate (in terms of u).

$$\tfrac{1}{3}\int \cos(u)\,du = \tfrac{1}{3}\left[\sin(u)\right] + C = \tfrac{1}{3}\sin(u) + C$$

5. Re-express in terms of the original variable x.

$$\int \cos\left(9x^2+15x+2\right)(6x+5)\,dx = \underbrace{\frac{1}{3}\sin\left(9x^2+15x+2\right)+C}_{\frac{1}{3}\sin(u)+C}$$

$$\text{i.e., } \int \cos\left(9x^2 + 15x + 2\right)(6x + 5)\, dx = \frac{1}{3}\sin\left(9x^2 + 15x + 2\right) + C$$

5. **Compute:** $\int_{-1}^{2} \left(6x^2 + 6x + 4\right) dx =$

$$\int_{-1}^{2} \underbrace{\left(6x^2 + 6x + 4\right)}_{f(x)} dx = \underbrace{\left[6\left(\frac{x^3}{3}\right) + 6\left(\frac{x2}{2}\right) + 4x\right]_{-1}^{2}}_{F(x)} = \underbrace{\left[2x^3 + 3x^2 + 4x\right]_{-1}^{2}}$$

$$= \underbrace{\left[2\left(2\right)^3 + 3\left(2\right)^2 + 4\left(2\right)\right]}_{F(2)} - \underbrace{\left[2\left(-1\right)^3 + 3\left(-1\right)^2 + 4\left(-1\right)\right]}_{F(-1)}$$

$$= 36 - (-3) = 39$$

$$\text{i.e., } \int_{-1}^{2} \left(6x^2 + 6x + 4\right) dx = 39$$

6. **Compute:** $\int_0^1 \left(3x^2-2x+1\right)^3 (3x-1)\ dx =$ (The answer may not be a whole number or a positive number)

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(3x^2-2x+1\right)^3$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(3x^2-2x+1\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(3x^2-2x+1\right)}_{\text{function}} ----\rightarrow \underbrace{(3x-1)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(3x^2-2x+1\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= \left(3x^2-2x+1\right) \\ \Rightarrow \frac{du}{dx} &= 6x-2 \\ \Rightarrow du &= (6x-2)\ dx \\ \Rightarrow \tfrac{1}{2}du &= (3x-1)\ dx \end{aligned}$$

When $x=0$, $u = 3x^2-2x+1 = 3\left(0\right)^2 - 2\left(0\right)+1 = 1$
When $x=1$, $u = 3x^2-2x+1 = 3\left(1\right)^2 - 2\left(1\right)+1 = 2$

3. Analyze in terms of u and du

$$\int_{x=0}^{x=1} \underbrace{\left(3x^2-2x+1\right)^3}_{u^3} \underbrace{(3x-1)\ dx}_{\frac{1}{2}du} = \int_{u=1}^{u=2} u^3 \cdot \tfrac{1}{2}du = \tfrac{1}{2}\int_{u=1}^{u=2} u^3 du$$

Don't forget to re-write the limits of integration in terms of u!

4. Integrate (in terms of u).

$$\frac{1}{2}\int_{u=1}^{u=2} u^3 du = \frac{1}{2}\left[\frac{u^4}{4}\right]_{u=1}^{u=2} = \left[\frac{u^4}{8}\right]_{u=1}^{u=2} = \underbrace{\frac{(2)^4}{8}}_{F(2)} - \underbrace{\frac{(1)^4}{8}}_{F(1)} = \frac{16}{8} - \frac{1}{8} = \frac{15}{8}$$

i.e., $\int_0^1 \left(3x^2 - 2x + 1\right)^3 \left(3x - 1\right)\, dx = \frac{15}{8}$