MTH 3318 Test #1

Spring 2024

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Instructions. Document your work fully.

For problems 1- 2 prove one using Mathematical Induction.

1. $2+4+6+\ldots+2n=n^2+n$
   i.e. $\sum_{i=1}^{n} 2i = n^2+n$

2. $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$
   i.e. $\sum_{j=1}^{n} \frac{1}{(2j-1)(2j+1)} = \frac{n}{2n+1}$

For problems 3- 5 prove one using Mathematical Induction.

3. $2 + 6 + 10 + \ldots + 4n - 2 = 2n^2$
   i.e. $\sum_{i=1}^{n} (4i - 2) = 2n^2$

4. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$
   i.e. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

5. $\frac{n^4}{4} < 1^3 + 2^3 + 3^3 + \ldots + n^3$ all natural numbers, n.

For problems 6 - 7, prove one using Mathematical Induction:

6. For $0 \leq a \leq b$; prove that $a^n \leq b^n$.

7. Given that $\frac{d}{dx}\left[x^0\right] = 0$ and $\frac{d}{dx}\left[x^1\right] = 1$, prove that $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$. You may use the product rule: $\frac{d}{dx}\left[f(x)\,g(x)\right] = f'(x)\,g(x) + g'(x)\,f(x)$.

For problems 8 - 9, prove one using Mathematical Induction:

8. $(1+x)^n \geq 1+nx$ for any natural number n and any real number $x \geq -1$.

9. Given that $|x_1+x_2| \leq |x_1|+|x_2|$ (the Triangle Inequality); Prove by induction that: $|x_1+x_2+x_3+\ldots+x_n| \leq |x_1|+|x_2|+|x_3|+\ldots+|x_n|$ (the General Triangle Inequality).