MTH 1125 1pm Class - Test #4 - Solutions

Fall 2018

Pat Rossi Name ________________________________

Show CLEARLY how you arrive at your answers!

1. **Compute:** $\int \left(12x^3 + 9x^2 + 6x + 3 + \sqrt{x}\right) dx =$

$\int \left(12x^3 + 9x^2 + 6x + 3 + \sqrt{x}\right) dx = \int \left(12x^3 + 9x^2 + 6x + 3 + x^{\frac{1}{2}}\right) dx$

$= 12\left[\frac{x^4}{4}\right] + 9\left[\frac{x^3}{3}\right] + 6\left[\frac{x^2}{2}\right] + 3x + \frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C = 3x^4 + 3x^3 + 3x^2 + 3x + \frac{2}{3}x^{\frac{3}{2}} + C$

i.e., $\int \left(12x^3 + 9x^2 + 6x + 3 + \sqrt{x}\right) dx = 3x^4 + 3x^3 + 3x^2 + 3x + \frac{2}{3}x^{\frac{3}{2}} + C$

2. **Compute:** $\int \left(6x^2+8x\right)^9 (3x+2)\, dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(6x^2+8x\right)^9$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(6x^2+8x\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(6x^2+8x\right)}_{\text{function}} - - - - - \rightarrow \underbrace{(3x+2)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(6x^2+8x\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{array}{rcl} u & = & 6x^2+8x \\ \Rightarrow \frac{du}{dx} & = & 12x+8 \\ \Rightarrow du & = & (12x+8)\, dx \\ \Rightarrow \frac{1}{4} du & = & (3x+2)\, dx \end{array}$$

3. Analyze in terms of u and du

$$\int \underbrace{\left(6x^2+8x\right)^9}_{u^9} \underbrace{(3x+2)\, dx}_{\frac{1}{4}du} = \int u^9 \tfrac{1}{4} du = \tfrac{1}{4} \int u^9 du$$

4. Integrate (in terms of u).

$$\tfrac{1}{4} \int u^9 du = \tfrac{1}{4} \left[\tfrac{u^{10}}{10}\right] + C = \tfrac{1}{40} u^{10} + C$$

5. Re-express in terms of the original variable, x.

$$\int \left(6x^2+8x\right)^9 (3x+2)\, dx = \underbrace{\frac{1}{40} \left(6x^2+8x\right)^{10} + C}_{\frac{1}{40}u^{10}+C}$$

i.e., $\int \left(6x^2+8x\right)^9 (3x+2)\, dx = \frac{1}{40} \left(6x^2+8x\right)^{10} + C$

3. **Compute:** $\int (3\sin(x) + 4\csc(x)\cot(x))\,dx =$

$\int (3\sin(x) + 4\csc(x)\cot(x))\,dx = 3\left[-\cos(x)\right] + 4\left[-\csc(x)\right] + C$

$= -3\cos(x) - 4\csc(x) + C$

i.e., $\int (3\sin(x) + 4\csc(x)\cot(x))\,dx = -3\cos(x) - 4\csc(x) + C$

4. **Compute:** $\int \cos\left(4x^2+12x+4\right)(2x+3)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\cos\left(4x^2+12x+4\right)$

   $\nearrow$ outer $\quad\nwarrow$ inner

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(4x^2+12x+4\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(4x^2+12x+4\right)}_{\text{function}} - - - - \rightarrow \underbrace{(2x+3)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(4x^2+12x+4\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 4x^2+12x+4 \\ \Rightarrow \frac{du}{dx} &= 8x+12 \\ \Rightarrow du &= (8x+12)\,dx \\ \Rightarrow \tfrac{1}{4}du &= (2x+3)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\cos\left(4x^2+12x+4\right)}_{\cos(u)}\underbrace{(2x+3)\,dx}_{\frac{1}{4}du} = \int \cos(u)\ \tfrac{1}{4}du = \tfrac{1}{4}\int \cos(u)\,du$$

4. Integrate (in terms of u).

$$\tfrac{1}{4}\int \cos(u)\,du = \tfrac{1}{4}\left[\sin(u)\right] + C = \tfrac{1}{4}\sin(u) + C$$

5. Re-express in terms of the original variable, x.

$$\int \cos\left(4x^2+12x+4\right)(2x+3)\,dx = \underbrace{\frac{1}{4}\sin\left(4x^2+12x+4\right)+C}_{\frac{1}{4}\sin(u)+C}$$

i.e., $\int \cos\left(4x^2+12x+4\right)(2x+3)\,dx = \frac{1}{4}\sin\left(4x^2+12x+4\right)+C$

5. **Compute:** $\int_{-1}^{1}\left(3x^2+4x+5\right)dx =$

$$\int_{-1}^{1}\underbrace{\left(3x^2+4x+5\right)}_{f(x)}dx = \underbrace{\left[3\frac{x^3}{3}+4\frac{x^2}{2}+5x\right]_{-1}^{1}}_{F(x)} = \underbrace{\left[x^3+2x^2+5x\right]_{-1}^{1}}_{F(x)} =$$

$$= \underbrace{\left[(1)^3+2\,(1)^2+5\,(1)\right]}_{F(1)} - \underbrace{\left[(-1)^3+2\,(-1)^2+5\,(-1)\right]}_{F(-1)} = 8-(-4) = 12$$

i.e., $\int_{-1}^{1}\left(3x^2+4x+5\right)dx = 12$

6. **Compute:** $\int_0^1 \left(2x^2+1\right)^3 x\;dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(2x^2+1\right)^3$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(2x^2+1\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(2x^2+1\right)}_{\text{function}} - - - - - \rightarrow \underbrace{x}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(2x^2+1\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 2x^2+1 \\ \Rightarrow \frac{du}{dx} &= 4x \\ \Rightarrow du &= 4x\;dx \\ \Rightarrow \frac{1}{4}du &= x\;dx \end{aligned}$$

   When $x=0$, $u = 2x^2+1 = 2\left(0\right)^2+1 = 1$
   When $x=1$, $u = 2x^2+1 = 2\left(1\right)^2+1 = 3$

3. Analyze in terms of u and du

$$\int_{x=0}^{x=1} \underbrace{\left(2x^2+1\right)^3}_{u^3} \underbrace{x\;dx}_{\frac{1}{4}du} = \int_{u=1}^{u=3} u^3 \cdot \frac{1}{4}du = \frac{1}{4}\int_{u=1}^{u=3} u^3 du$$

   Don't forget to re-write the limits of integration in terms of u!

4. Integrate (in terms of u).

$$\frac{1}{4}\int_{u=1}^{u=3} u^3 du = \frac{1}{4}\left[\frac{u^4}{4}\right]_{u=1}^{u=3} = \left[\frac{u^4}{16}\right]_{u=1}^{u=3} = \underbrace{\frac{\left(3\right)^4}{16}}_{F(3)} - \underbrace{\frac{\left(1\right)^4}{16}}_{F(1)} = \frac{81}{16} - \frac{1}{16} = 5$$

   i.e., $\int_{x=0}^{x=1} \left(2x^2+1\right)^3 x\;dx = 5$

7. **Compute:** $\frac{d}{dx}\left[\ln\left(\sin\left(x\right)\right)\right] =$

$$\underbrace{\frac{d}{dx}\left[\ln\left(\sin\left(x\right)\right)\right]}_{\frac{d}{dx}[\ln(g(x))]} = \underbrace{\frac{1}{\sin\left(x\right)}}_{\frac{1}{g(x)}} \cdot \underbrace{\cos\left(x\right)}_{g'(x)} = \frac{\cos(x)}{\sin(x)} = \cot\left(x\right)$$

$$\boxed{\text{i.e., } \frac{d}{dx}\left[\ln\left(\sin\left(x\right)\right)\right] = \frac{\cos(x)}{\sin(x)} = \cot\left(x\right)}$$

8. **Compute:** $\int \frac{\cos(x)}{\sin(x)+5}dx =$

$$\int \frac{\cos(x)}{\sin(x)+5}dx \underbrace{=}_{\text{re-write}} \int \frac{1}{\sin(x)+5}\ \cos(x)\ dx$$

Remark: Note that we have an approximate function/derivative pair, with the "function" in the denominator. This usually indicates that u-substitution is appropriate, with the result being a natural logarithm.

1. Is u-sub appropriate?

   a. Is there a composite function?

      Yes! $\frac{1}{\sin(x)+5}$ is the same as $(\sin(x)+5)^{-1}$, so it is a function raised to a power.

      Let $u =$ the "inner" of the composite function

      $\Rightarrow u = \sin(x) + 5$

   b. Is there an (approximate) function/derivative pair?

      Yes! $\underbrace{(\sin(x)+5)}_{\text{function}} - - - - \rightarrow \underbrace{\cos(x)}_{\text{deriv}}$

      Let $u =$ the "function" of the function/deriv pair

      $\Rightarrow u = \sin(x) + 5$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

      (i.e., do criteria **a** and **b** suggest the same choice of u ?)

      Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= \sin(x) + 5 \\ \Rightarrow \frac{du}{dx} &= \cos(x) \\ \Rightarrow du &= \cos(x)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\underbrace{\int \frac{1}{\sin(x)+5}}_{\frac{1}{u}} \underbrace{\cos(x)\ dx}_{du} = \int \frac{1}{u} du$$

4. Integrate (in terms of u).

   $\int \frac{1}{u} du = \ln|u| + C$

5. Re-express in terms of the original variable, x.

$$\int \frac{\cos(x)}{\sin(x)+5}dx = \underbrace{\ln|\sin(x)+5| + C}_{\ln|u|+C}$$

i.e., $\int \frac{\cos(x)}{\sin(x)+5}dx = \ln|\sin(x)+5| + C$