MTH 3318 Test #1

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Instructions. Fully document your work.

For problems 1 - 2, prove one using Mathematical Induction:

1. For $0 \leq a \leq b$; prove that $a^n \leq b^n$.

2. Given that $\frac{d}{dx}\left[x^0\right] = 0$ and $\frac{d}{dx}\left[x^1\right] = 1$, prove that $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$. You may use the product rule: $\frac{d}{dx}\left[f(x)\,g(x)\right] = f'(x)\,g(x) + g'(x)\,f(x)$.

For problems 3 - 4, prove one using Mathematical Induction:

3. $(1+x)^n \geq 1+nx$ for any natural number n and any real number $x \geq -1$.

4. Given that $|x_1 + x_2| \leq |x_1| + |x_2|$ (the Triangle Inequality); Prove by induction that: $|x_1 + x_2 + x_3 + \ldots + x_n| \leq |x_1| + |x_2| + |x_3| + \ldots + |x_n|$ (the General Triangle Inequality).

For problems 5- 6 prove one using Mathematical Induction.

5. $2 + 4 + 6 + \ldots + 2n = n^2 + n$
   i.e. $\sum_{i=1}^{n} 2i = n^2 + n$

6. $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$
   i.e. $\sum_{j=1}^{n} \frac{1}{(2j-1)(2j+1)} = \frac{n}{2n+1}$

For problems 7- 9 prove one using Mathematical Induction.

7. $2 + 6 + 10 + \ldots + 4n - 2 = 2n^2$
   i.e. $\sum_{i=1}^{n} (4i - 2) = 2n^2$

8. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$
   i.e. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

9. $\frac{n^4}{4} < 1^3 + 2^3 + 3^3 + \ldots + n^3$ all natural numbers, n.

10. In exercises 10.a - 10.d, let p be the statement: "He practices every night," and let q be the statement: "he will achieve greatness." Write each statement in symbolic form.

    (a) If he practices every night, then he will achieve greatness.

    (b) He will practice every night, or he will not achieve greatness.

    (c) His practicing every night is a necessary and sufficient condition for him to achieve greatness.

    (d) He will have practiced every night if he achieves greatness.

11. In exercises 7 - 9, let p be the statement: "Fall came early this year," and let q be the statement: "I love the weather." Write each statement in words.

    (a) $p \wedge q$

    (b) $p \vee q$

    (c) $q \rightarrow \sim p$

    (d) $\sim p \leftrightarrow \sim q$

12. In problems 12.a - 12.d, determine whether the given propositions are True or False:

    (a) If $8 + 3 = 9$, then $7 < 10$.

    (b) If $8 > 3$, then $8 > 5$.

    (c) $8 > 10$ if and only if $2 + 2 = 5$.

    (d) If $2 + 2 = 4$, then $8 > 10$.

13. In exercises 13.a-13.b construct a truth table for the statement given.

    (a) $p \longleftrightarrow (q \wedge r)$

    (b) $(\sim p \vee q) \rightarrow r$