MTH 1125 - Test 2 (2pm Class) - Solutions

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\frac{d}{dx}\left[4x^5+6x^4+9x^3+12x^2+20x+16\sqrt{x}+2\right]=$

$\frac{d}{dx}\left[4x^5+6x^4+9x^3+12x^2+20x+16\sqrt{x}+2\right]$

$=4\left[5x^4\right]+6\left[4x^3\right]+9\left[3x^2\right]+12\left[2x\right]+20+16\left[\frac{1}{2}x^{-\frac{1}{2}}\right]+0$

$=20x^4+24x^3+27x^2+24x+20+8x^{-\frac{1}{2}}$

i.e., $\frac{d}{dx}\left[4x^5+6x^4+9x^3+12x^2+20x+16\sqrt{x}+2\right]=20x^4+24x^3+27x^2+24x+20+8x^{-\frac{1}{2}}$

2. Compute: $\frac{d}{dx}\left[\left(x^4+4x\right)\sec(x)\right]=$

$$\frac{d}{dx}\left[\underbrace{\left(x^4+4x\right)}_{1^{st}}\underbrace{\sec(x)}_{2^{nd}}\right]=\underbrace{\left(4x^3+4\right)}_{1^{\text{st}}\text{ prime}}\cdot\underbrace{\sec(x)}_{2^{nd}}+\underbrace{\sec(x)\tan(x)}_{2^{\text{nd}}\text{ prime}}\cdot\underbrace{\left(x^4+4x\right)}_{1^{st}}$$

$\frac{d}{dx}\left[\left(x^4+4x\right)\sec(x)\right]=\left(4x^3+4\right)\sec(x)+\sec(x)\tan(x)\left(x^4+4x\right)$

3. Compute: $\frac{d}{dx}\left[\frac{4x^2+3x+3}{3x^2-6x+2}\right]=$

$$\frac{d}{dx}\left[\frac{\overbrace{4x^2+3x+3}^{\text{top}}}{\underbrace{3x^2-6x+2}_{\text{Bottom}}}\right]=\frac{\overbrace{(8x+3)}^{\text{top prime}}\cdot\overbrace{\left(3x^2-6x+2\right)}^{\text{bottom}}-\overbrace{(6x-6)}^{\text{bottom prime}}\cdot\overbrace{\left(4x^2+3x+3\right)}^{\text{top}}}{\underbrace{\left(3x^2-6x+2\right)^2}_{\text{bottom squared}}}$$

i.e., $\frac{d}{dx}\left[\frac{4x^2+3x+3}{3x^2-6x+2}\right]=\frac{(8x+3)\left(3x^2-6x+2\right)+(6x-6)\left(4x^2+3x+3\right)}{\left(3x^2-6x+2\right)^2}$

4. Compute: $\frac{d}{dx}\left[\left(6x^5+\sec(x)\right)^{10}\right] =$ This is the derivative of a function raised to a power.

$$\frac{d}{dx}\left[\left(6x^5+\sec(x)\right)^{10}\right] = \underbrace{10\left(6x^5+\sec(x)\right)^9}_{\substack{\text{power rule} \\ \text{as usual}}} \cdot \underbrace{\left(30x^4+\sec(x)\tan(x)\right)}_{\substack{\text{derivative} \\ \text{of inner}}}$$

i.e., $\frac{d}{dx}\left[\left(6x^5+\sec(x)\right)^{10}\right] = 10\left(6x^5+\sec(x)\right)^9\left(30x^4+\sec(x)\tan(x)\right)$

5. Given that $f(x)=2x^2+4x+3$, give the *equation* of the line tangent to the graph of $f(x)$ at the point $(2,19)$.

We need two things:

i. A point on the line (We have that: $(x_1,y_1)=(2,19)$)

ii. The slope of the line (This is $f'(x_1)$)

$f'(x)=4x+4$

At the point $(x_1,y_1)=(2,19)$, **the slope is** $f'(2)=4(2)+4=12$

We will use the Point-Slope equation of a line:

$y-y_1=m(x-x_1)$ (Where m is the slope and (x_1,y_1) is a known point on the line.)

Thus, the equation of the line tangent to the graph of $f(x)$ is:

$y-19=12(x-2)$

The equation of the line tangent is $y-19=12(x-2)$

6. Given that $y = \csc(t)$ and that $t = \frac{1}{2}x^2 + 4x$; compute $\frac{dy}{dx}$ **using the Liebniz form of the Chain Rule.** (In particular, when doing this exercise, *write explicitly the Liebniz form of the chain rule that you are going to use.)*

We know:

$\frac{dy}{dt} = -\csc(t)\cot(t)$

$\frac{dt}{dx} = x + 4$

We want: $\frac{dy}{dx}$

By the Liebniz form of the Chain Rule:

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = -\csc(t)\cot(t)(x+4) = \underbrace{-\csc\left(\frac{1}{2}x^2 + 4x\right)\cot\left(\frac{1}{2}x^2 + 4x\right)(x+4)}_{\substack{\text{express solely in terms of} \\ \text{independent variable } x}}$$

$$\boxed{\text{i.e. } \frac{dy}{dx} = -\csc\left(\tfrac{1}{2}x^2 + 4x\right)\cot\left(\tfrac{1}{2}x^2 + 4x\right)(x+4)}$$

7. Compute: $\frac{d}{dx}\left[\cos\left(5x^3 + 8x^2 + 3\right)\right] =$

Outer:	$=$	$\cos(\quad)$
Deriv. of outer	$=$	$-\sin(\quad)$

$$\frac{d}{dx}\left[\begin{array}{l} \cos\left(\underbrace{5x^3 + 8x^2 + 3}\right) \\ \uparrow \qquad\quad \uparrow \\ \text{outer} \quad \text{inner} \end{array}\right] = \underbrace{-\sin\left(5x^3 + 8x^2 + 3\right)}_{\substack{\text{Deriv of outer,} \\ \text{eval. at inner}}} \cdot \underbrace{\left(15x^2 + 16x\right)}_{\substack{\text{deriv. of} \\ \text{inner}}}$$

$$\boxed{\text{i.e., } \frac{d}{dx}\left[\cos\left(5x^3 + 8x^2 + 3\right)\right] = -\sin\left(5x^3 + 8x^2 + 3\right)\left(15x^2 + 16x\right)}$$

8. Compute: $\frac{d}{dx}\left[\left(\frac{3x^4+12x}{2x^4+8x}\right)^6\right] =$ In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE.

$$\frac{d}{dx}\left[\underbrace{\left(\frac{3x^4+12x}{2x^4+8x}\right)^6}_{(g(x))^n}\right] = \underbrace{6\left(\frac{3x^4+12x}{2x^4+8x}\right)^5}_{\substack{\text{power rule}\\\text{as usual}}} \cdot \underbrace{\left(\frac{d}{dx}\left[\frac{3x^4+12x}{2x^4+8x}\right]\right)}_{\substack{\text{deriv of}\\\text{inner Function}}}$$

$$= 6\left(\frac{3x^4+12x}{2x^4+8x}\right)^5 \underbrace{\frac{(12x^3+12)(2x^4+8x) - (8x^3+8)(3x^4+12x)}{(2x^4+8x)^2}}_{\substack{\text{quotient}\\\text{rule}}}$$

i.e., $\frac{d}{dx}\left[\left(\frac{3x^4+12x}{2x^4+8x}\right)^6\right] = 6\left(\frac{3x^4+12x}{2x^4+8x}\right)^5 \frac{(12x^3+12)(2x^4+8x) - (8x^3+8)(3x^4+12x)}{(2x^4+8x)^2}$

9. Compute: $\frac{d}{dx}\left[\csc^5(2x^4+8x)\right] =$ In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE. Re-write!

$\frac{d}{dx}\left[(\csc(2x^4+8x))^5\right]$ This is the derivative of a function, raised to a power

$$\frac{d}{dx}\left[(\csc(2x^4+8x))^5\right] = \underbrace{5\left(\csc\left(2x^4+8x\right)\right)^4}_{\substack{\text{power rule}\\\text{as usual}}} \cdot \underbrace{\left(\frac{d}{dx}\csc\left(2x^4+8x\right)\right)}_{\substack{\text{derivative}\\\text{of inner}}}$$

$$= 5(\csc(2x^4+8x))^4 \cdot \underbrace{\left[-\csc\left(2x^4+8x\right)\cot\left(2x^4+8x\right)\cdot\left(8x^3+8\right)\right]}_{\substack{\text{Chain}\\\text{Rule}}}$$

$$= -5(\csc(2x^4+8x))^4 \cdot \underbrace{\csc\left(2x^4+8x\right)\cot\left(2x^4+8x\right)\cdot\left(8x^3+8\right)}_{\substack{\text{Chain}\\\text{Rule}}}$$

i.e., $\frac{d}{dx}\left[(\csc(2x^4+8x))^5\right] = -5(\csc(2x^4+8x))^4\csc(2x^4+8x)\cot(2x^4+8x)(8x^3+8)$

10. Given that $L'(x) = \frac{1}{x}$ (i.e., $\frac{d}{dx}[L(x)] = \frac{1}{x}$); compute $\frac{d}{dx}[L(x^3)]$

Outer: $=$	$L(\quad)$
Deriv. of outer $=$	$\frac{1}{(\quad)}$

$$\frac{d}{dx}\Big[\underbrace{L}_{\uparrow\ \text{outer}}\Big(\underbrace{x^3}_{\uparrow\ \text{inner}}\Big)\Big] = \underbrace{\frac{1}{x^3}}_{\text{Deriv of outer, eval. at inner}} \cdot \underbrace{3x^2}_{\text{deriv. of inner}} = \frac{3x^2}{x^3} = \frac{3}{x}$$

i.e., $\frac{d}{dx}[L(x^3)] = \frac{3x^2}{x^3} = \frac{3}{x}$

11. Given that $f(x) = 4x^2 - 3x + 5$, compute $f'(x)$ **using the definition of derivative.** (i.e., using the "limit process.")

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{[4(x+\Delta x)^2 - 3(x+\Delta x)+] - [4x^2 - 3x + 5]}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{[4(x^2 + 2x\Delta x + \Delta x^2) - 3(x+\Delta x) + 5] - [4x^2 - 3x + 5]}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{[4x^2 + 8x\Delta x + 4\Delta x^2 - 3x - 3\Delta x + 5] - [4x^2 - 3x + 5]}{\Delta x} = \lim_{\Delta x \to 0} \frac{8x\Delta x + 4\Delta x^2 - 3\Delta x}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\Delta x(8x + 4\Delta x - 3)}{\Delta x} = \lim_{\Delta x \to 0} (8x + 4\Delta x - 3) = 8x + 4(0) - 3 = 8x - 3$

i.e., $f'(x) = 8x - 3$

12. Given that $x^4 + y^4 = \sin(y)$, compute y'

i. Differentiate both sides w.r.t. x

$\frac{d}{dx}\left[x^4 + y^4\right] = \frac{d}{dx}\left[\sin(y)\right]$

$\Rightarrow 4x^3 + 4y^3 \cdot y' = \cos(y) \cdot y'$

ii. Solve algebraically for y'

a. Get y' terms on left side, all other terms on right side

$\Rightarrow 4y^3 \cdot y' - \cos(y) \cdot y' = -4x^3$

b. Factor out y'

$\Rightarrow (4y^3 - \cos(y))\, y' = -4x^3$

c. Divide both sides by the cofactor of y'

$y' = \frac{-4x^3}{4y^3 - \cos(y)} = -\frac{4x^3}{4y^3 - \cos(y)}$

$$\boxed{y' = -\frac{4x^3}{4y^3 - \cos(y)}}$$