

Logic Homework Exercises #4 (Arguments) - Solutions - Alternate Version

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Name _____

Instructions. Determine whether each argument is valid.

1. If I study hard, then I will get A's. I will study hard. Therefore, I will get A's.

Solution: If we make the assignment:

p : I study hard. q : I will get A's.

The argument is of the form:

If $\underbrace{\text{I study hard}}_p$, $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{\text{I will get A's.}}_q$ $\underbrace{\text{I will study hard.}}_p$. Therefore, $\underbrace{\text{I will get A's.}}_q$.

Our premises are:

$p_1 : p \rightarrow q$

$p_2 : p$

Our conclusion is:

$C : q$

So our argument has the form: $(p_1 \wedge p_2) \rightarrow C$

p	q	$p_1 : p \rightarrow q$	$p_2 : p$	$C : q$	$p_1 \wedge p_2$	$(p_1 \wedge p_2) \rightarrow C$
T	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	F	F	F	T

Since the argument is a tautology, the argument is VALID.

2. If I study hard, then I get A's. If I don't get rich, then I don't get A's. Therefore, I get rich.

Solution: If we make the assignment:

p : I study hard. q : I get A's. r : I get rich

The argument is of the form:

If I study hard, then I get A's. If I don't get rich, then I don't get A's. Therefore, I get rich.

$$\underbrace{p}_{p} \rightarrow \underbrace{q}_{q} \quad \underbrace{\sim r}_{\sim r} \rightarrow \underbrace{\sim q}_{\sim q} \quad \text{Therefore, } \underbrace{r}_{r}$$

Our premises are:

$p_1 : p \rightarrow q$

$p_2 : \sim r \rightarrow \sim q$

Our conclusion is:

$C : r$

So our argument has the form: $(p_1 \wedge p_2) \rightarrow C$

p	q	r	$\sim q$	$\sim r$	$p_1 : p \rightarrow q$	$p_2 : \sim r \rightarrow \sim q$	$C : r$	$(p_1 \wedge p_2)$	$(p_1 \wedge p_2) \rightarrow C$
T	T	T	F	F	T	T	T	T	T
T	T	F	F	T	T	F	F	F	T
T	F	T	T	F	F	T	T	F	T
T	F	F	T	T	F	T	F	F	T
F	T	T	F	F	T	T	T	T	T
F	T	F	F	T	T	F	F	F	T
F	F	T	T	F	T	T	T	T	T
F	F	F	T	T	T	T	F	T	F

Since the argument is not a tautology, the argument is INVALID.

3. I study hard, if and only if I get rich. I get rich. Therefore, I study hard.

Solution: If we make the assignment:

p : I study hard. q : I get rich,

The argument is of the form:

$\underbrace{\text{I study hard}}_p$ \leftrightarrow $\underbrace{\text{if and only if}}_{\leftrightarrow}$ $\underbrace{\text{I get rich}}_q$ \wedge $\underbrace{\text{I get rich}}_q$ Therefore, $\underbrace{\text{I study hard}}_p$.

Our premises are:

$p_1 : p \leftrightarrow q$

$p_2 : q$

Our conclusion is:

$C : p$

So our argument has the form: $(p_1 \wedge p_2) \rightarrow C$

p	q	$p_1 : p \leftrightarrow q$	$p_2 : q$	$C : p$	$(p_1 \wedge p_2)$	$(p_1 \wedge p_2) \rightarrow C$
T	T	T	T	T	T	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	T	F	F	F	T

Since the argument is a tautology, the argument is VALID.

4. If I study hard or I get rich, then I get A's. I get A's. Therefore, if I don't study hard, then I get rich.

Solution: If we make the assignment:

p : I study hard. q : I get rich r : I get A's

The argument is of the form:

If $\underbrace{\text{I study hard}}_p$ \vee $\underbrace{\text{I get rich}}_q$, then $\underbrace{\text{I get A's}}_r$. $\underbrace{\text{I get A's}}_r$.

Therefore, if $\underbrace{\text{I don't study hard}}_{\sim p}$, then $\underbrace{\text{I get rich}}_q$.

Our premises are:

$p_1 : (p \vee q) \rightarrow r$

$p_2 : r$

Our conclusion is:

$C : \sim p \rightarrow q$

So our argument has the form: $(p_1 \wedge p_2) \rightarrow C$

p	q	r	$\sim p$	$p \vee q$	$p_1 : (p \vee q) \rightarrow r$	$p_2 : r$	$p_1 \wedge p_2$	$C : \sim p \rightarrow q$	$(p_1 \wedge p_2) \rightarrow C$
T	T	T	F	T	T	T	T	T	T
T	T	F	F	T	F	F	F	T	T
T	F	T	F	T	T	T	T	T	T
T	F	F	F	T	F	F	F	T	T
F	T	T	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F	T	T
F	F	T	T	F	T	T	T	F	F
F	F	F	T	F	T	F	F	F	T

Since the argument is not a tautology, the argument is INVALID.

5. If I study hard, then I get A's or I get rich. I don't get A's and I don't get rich. Therefore, I don't study hard.

Solution: If we make the assignment:

p : I study hard. q : I get A's r : I get rich

The argument is of the form:

If $\underbrace{\text{I study hard.}}_p$, then $\underbrace{(\text{I get A's})}_q$ \vee $\underbrace{\text{I get rich.}}_r$. $\underbrace{\text{I don't get A's}}_{\sim q}$ and $\underbrace{\text{I don't get rich.}}_{\sim r}$.

Therefore, $\underbrace{\text{I don't study hard.}}_{\sim p}$.

Our premises are:

$p_1 : p \rightarrow (q \vee r)$

$p_2 : \sim q \wedge \sim r$

Our conclusion is:

$C : \sim p$

So our argument has the form: $(p_1 \wedge p_2) \rightarrow C$

p	q	r	$\sim p$	$\sim q$	$\sim r$	$(q \vee r)$	$p_1 : p \rightarrow (q \vee r)$	$p_2 : \sim q \wedge \sim r$	$p_1 \wedge p_2$	$C : \sim p$	$(p_1 \wedge p_2) \rightarrow C$
T	T	T	F	F	F	T	T	F	F	F	T
T	T	F	F	F	T	T	T	F	F	F	T
T	F	T	F	T	F	T	T	F	F	F	T
T	F	F	F	T	T	F	F	T	F	F	T
F	T	T	T	F	F	T	T	F	F	T	T
F	T	F	T	F	T	T	T	F	F	T	T
F	F	T	T	T	F	T	T	F	F	T	T
F	F	F	T	T	T	F	T	T	T	T	T

Since the argument is a tautology, the argument is VALID.