MTH 4441 Homework #3 - Part 1

DUE: MONDAY, SEPTEMBER 20, 2021

Pat Rossi Name _____

Def - Let $n \geq 2$ be a natural number. Two integers a and b are congruent modulo n, denoted $a \equiv b \pmod{n}$, exactly when $a - b = kn$, for some integer, k. Otherwise, a is incongruent to b modulo n, denoted $a \not\equiv b \pmod{n}$.

Alternative Definition - Let $n \geq 2$ be a natural number. For arbitrary integers a and b, $a \equiv b \pmod{n}$ iff a and b have the same remainder (by division algorithm) when divided by n.

Theorem 1 *Let $n \geq 2$ be fixed. Then for arbitrary integers:*

- $a \equiv a \pmod{n}$
- *If* $a \equiv b \pmod{n}$, *then* $b \equiv a \pmod{n}$
- *If* $a \equiv b \pmod{n}$ *and* $b \equiv c \pmod{n}$, *then* $a \equiv c \pmod{n}$
- *If* $a \equiv b \pmod{n}$ *and* $c \equiv d \pmod{n}$, *then* $a + c \equiv b + d \pmod{n}$ *and* $ac \equiv bd \pmod{n}$
- *If* $a \equiv b \pmod{n}$ *then* $a + c \equiv b + c \pmod{n}$ *and* $ac \equiv bc \pmod{n}$
- *If* $a \equiv b \pmod{n}$ *then* $a^k \equiv b^k \pmod{n}$

Compute the Following:

In each case, the answer should be the integer between 0 and $n - 1$ to which the given expression is congruent modulo n.

1. $32 \pmod{3} \equiv$
2. $27 \pmod{5} \equiv$
3. $81 \pmod{7} \equiv$
4. $81 \pmod{6} \equiv$
5. $81 \pmod{12} \equiv$
6. $56 \pmod{9} \equiv$
7. $59 \pmod{6} \equiv$

8. $59 \pmod 7 \equiv$

9. $59^5 \pmod 7 \equiv$

10. $59^5 \pmod 6 \equiv$

11. $(45 \cdot 54) \pmod 4 \equiv$

12. $(253 \cdot 146) \pmod 5 \equiv$