# MTH 1125 - Test 2 (12pm Class) - Solutions

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**Instructions.** Show CLEARLY how you arrive at your answers.

1. Compute: $\frac{d}{dx}\left[3x^5+4x^4+6x^3+9x^2+16x+30\sqrt{x}+10\right]=$

$\frac{d}{dx}\left[3x^5+4x^4+6x^3+9x^2+16x+30x^{\frac{1}{2}}+10\right]$

$=3\left[5x^4\right]+4\left[4x^3\right]+6\left[3x^2\right]+9\left[2x\right]+16+30\left[\frac{1}{2}x^{-\frac{1}{2}}\right]+0$

$=15x^4+16x^3+18x^2+18x+16+15x^{-\frac{1}{2}}$

i.e., $\frac{d}{dx}\left[3x^5+4x^4+6x^3+9x^2+16x+30\sqrt{x}+10\right]=15x^4+16x^3+18x^2+18x+16+15x^{-\frac{1}{2}}$

2. Compute: $\frac{d}{dx}\left[(\tan(x)+\sec(x))(5x^2+3x)\right]=$

$$\frac{d}{dx}\left[\underbrace{(\tan(x)+\sec(x))}_{1^{st}}\underbrace{(5x^2+3x)}_{2^{nd}}\right]=\underbrace{\left(\sec^2(x)+\sec(x)\tan(x)\right)}_{1^{\text{st}}\text{ prime}}\cdot\underbrace{(5x^2+3x)}_{2^{nd}}+\underbrace{(10x+3)}_{2^{\text{nd}}\text{ prime}}\cdot\underbrace{(\tan(x)+\sec(x))}_{1^{st}}$$

$\frac{d}{dx}\left[(\tan(x)+\sec(x))(5x^2+3x)\right]=\left(\sec^2(x)+\sec(x)\tan(x)\right)(5x^2+3x)+(10x+3)(\tan(x)+\sec(x))$

3. Compute: $\frac{d}{dx}\left[\frac{3x^4+6x^3+18x}{8x^2+8}\right]=$

$$\frac{d}{dx}\left[\frac{\overbrace{3x^4+6x^3+18x}^{\text{top}}}{\underbrace{8x^2+8}_{\text{Bottom}}}\right]=\frac{\overbrace{(12x^3+18x^2+18)}^{\text{top prime}}\cdot\overbrace{(8x^2+8)}^{\text{bottom}}-\overbrace{16x}^{\text{bottom prime}}\cdot\overbrace{(3x^4+6x^3+18x)}^{\text{top}}}{\underbrace{(8x^2+8)^2}_{\text{bottom squared}}}$$

i.e., $\frac{d}{dx}\left[\frac{3x^4+6x^3+18x}{8x^2+8}\right]=\frac{(12x^3+18x^2+18)(8x^2+8)-16x(3x^4+6x^3+18x)}{(8x^2+8)^2}$

4. Compute: $\frac{d}{dx}\left[\left(4x^3+8x^2+20x\right)^6\right] =$ This is the derivative of a function raised to a power.

$$\frac{d}{dx}\left[\left(4x^3+8x^2+20x\right)^6\right] = \underbrace{6\left(4x^3+8x^2+20x\right)^5}_{\substack{\text{power rule}\\ \text{as usual}}} \cdot \underbrace{\left(12x^2+16x+20\right)}_{\substack{\text{derivative}\\ \text{of inner}}}$$

> i.e., $\frac{d}{dx}\left[\left(4x^3+8x^2+20x\right)^6\right] = 6\left(4x^3+8x^2+20x\right)^5\left(12x^2+16x+20\right)$

5. Given that $f(x) = 5x^2+5x-5$, give the *equation* of the line tangent to the graph of $f(x)$ at the point $(1, 5)$.

We need two things:

i. A **point** on the line (We have that: $(x_1, y_1) = (1, 5)$)

ii. The **slope** of the line (This is $f'(x_1)$)

$f'(x) = 10x + 5$

At the point $(x_1, y_1) = (1, 5)$, **the slope is** $f'(1) = 10(1) + 5 = 15$

We will use the Point-Slope equation of a line:

$(y - y_1) = m(x - x_1)$ (Where $m$ is the slope and $(x_1, y_1)$ is a known point on the line.)

Thus, the equation of the line tangent to the graph of $f(x)$ is:

$(y - 5) = 15(x - 1)$

> The equation of the line tangent is $(y - 5) = 15(x - 1)$

6. Given that $w = \tan(u)$ and that $u = 3v^2 + 2v + 5$; compute $\frac{dw}{dv}$ **using the Liebniz form of the Chain Rule.** (In particular, when doing this exercise, *write explicitly the Liebniz form of the chain rule that you are going to use.)*

   **We know:**

   $\frac{dw}{du} = \sec^2(u)$

   $\frac{du}{dv} = 6v + 2$

   **We want:** $\frac{dw}{dv}$

   By the Liebniz form of the Chain Rule:

   $$\frac{dw}{dv} = \frac{dw}{du}\frac{du}{dv} = \sec^2(u)(6v+2) = \underbrace{\sec^2\left(3v^2+2v+5\right)(6v+2)}_{\substack{\text{express solely in terms of} \\ \text{independent variable } v}}$$

   | i.e. $\frac{dw}{dv} = \sec^2(3v^2+2v+5)(6v+2)$ |
   |---|

7. Compute: $\frac{d}{dx}\left[\sec\left(2x^3+3x^2\right)\right] =$

   | | | |
   |---|---|---|
   | Outer: | = | sec (   ) |
   | Deriv. of outer | = | sec (   ) tan (   ) |

   $$\frac{d}{dx}\left[\underset{\substack{\uparrow \\ \text{outer}}}{\sec}\left(\underset{\substack{\uparrow \\ \text{inner}}}{\underbrace{2x^3+3x^2}}\right)\right] = \underbrace{\sec\left(2x^3+3x^2\right)\tan\left(2x^3+3x^2\right)}_{\substack{\text{Deriv of outer,} \\ \text{eval. at inner}}} \cdot \underbrace{\left(6x^2+6x\right)}_{\substack{\text{deriv. of} \\ \text{inner}}}$$

   | i.e., $\frac{d}{dx}\left[\sec\left(2x^3+3x^2\right)\right] = \sec\left(2x^3+3x^2\right)\tan\left(2x^3+3x^2\right)\left(6x^2+6x\right)$ |
   |---|

8. Compute: $\frac{d}{dx}\left[\left(\frac{2x^2+6x}{5x^2+10x+15}\right)^5\right] =$ In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE.

$$\frac{d}{dx}\left[\underbrace{\left(\frac{2x^2+6x}{5x^2+10x+15}\right)^5}_{(g(x))^n}\right] = \underbrace{5\left(\frac{2x^2+6x}{5x^2+10x+15}\right)^4}_{\text{power rule as usual}} \cdot \underbrace{\left(\frac{d}{dx}\left[\frac{2x^2+6x}{5x^2+10x+15}\right]\right)}_{\text{deriv of inner Function}}$$

$$= 5\left(\frac{2x^2+6x}{5x^2+10x+15}\right)^4 \underbrace{\frac{(4x+6)\left(5x^2+10x+15\right) - (10x+10)\left(2x^2+6x\right)}{\left(5x^2+10x+15\right)^2}}_{\text{quotient rule}}$$

i.e., $\frac{d}{dx}\left[\left(\frac{2x^2+6x}{5x^2+10x+15}\right)^5\right] = 5\left(\frac{2x^2+6x}{5x^2+10x+15}\right)^4 \frac{(4x+6)\left(5x^2+10x+15\right) - (10x+10)\left(2x^2+6x\right)}{\left(5x^2+10x+15\right)^2}$

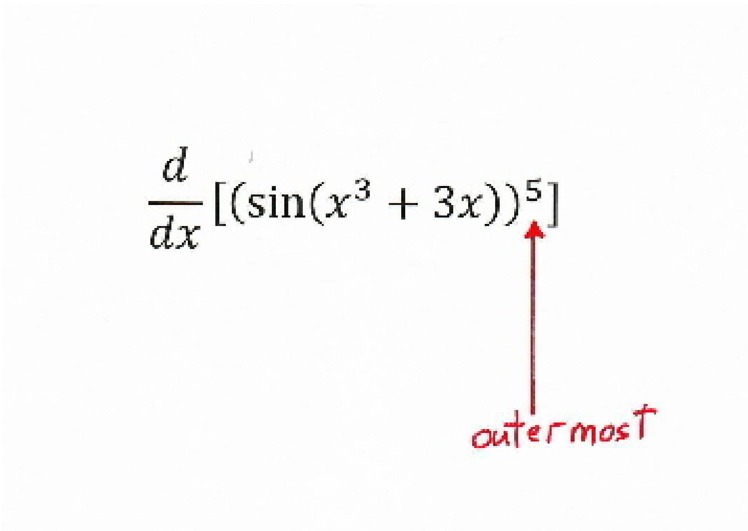
9. Compute: $\frac{d}{dx}\left[\sin^5\left(x^3+3x\right)\right] =$

Let's rewrite this:

$\frac{d}{dx}\left[\left(\sin\left(x^3+3x\right)\right)^5\right]$

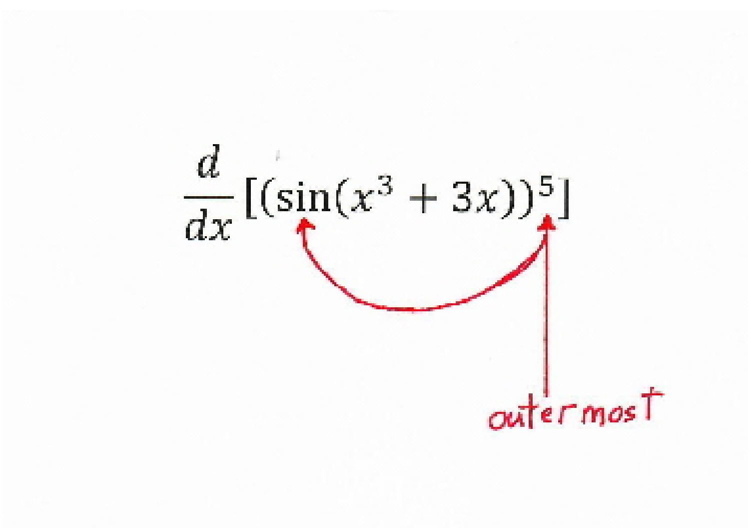
This is the composition of *three* functions.

Differentiate the outermost function and evaluate it at everything inside



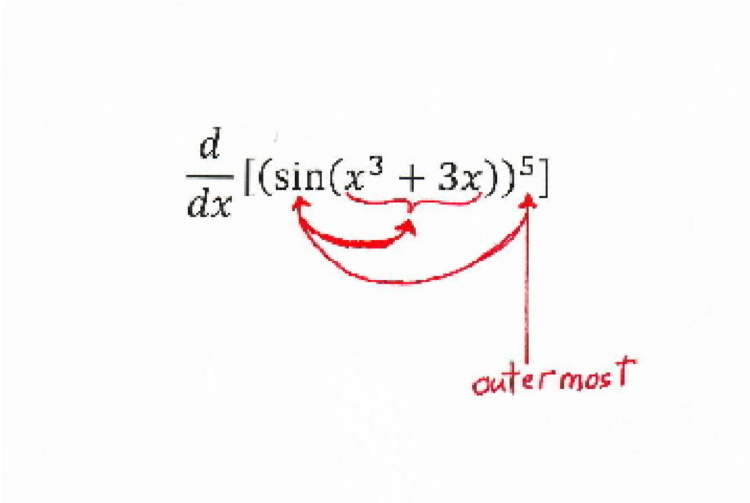

This yields: $5\left(\sin\left(x^3+3x\right)\right)^4$

**Next:** Multiply by the derivative of the next outermost function and evaluate it at everything inside of it.




This yields: $5\left(\sin\left(x^3+3x\right)\right)^4 \cdot \cos\left(x^3+3x\right)$

**Finally:** Multiply by the derivative of the innermost function.




This yields: $5\left(\sin\left(x^3+3x\right)\right)^4 \cdot \cos\left(x^3+3x\right) \cdot \left(3x^2+3x\right)$

i.e., $\frac{d}{dx}\left[\csc^{10}\left(4x^4+16x\right)\right] = 5\left(\sin\left(x^3+3x\right)\right)^4 \cdot \cos\left(x^3+3x\right) \cdot \left(3x^2+3x\right)$

**Alternatively:**

**Re-Write!**

$\frac{d}{dx}\left[\sin^5\left(x^3+3x\right)\right] = \frac{d}{dx}\left[\left(\sin\left(x^3+3x\right)\right)^5\right]$

In the broadest sense, this is *the derivative of a function raised to a power*

$$\frac{d}{dx}\left[\left(\sin\left(x^3+3x\right)\right)^5\right] = \underbrace{5\left(\sin\left(x^3+3x\right)\right)^4}_{\text{power rule as usual}} \cdot \underbrace{\left(\frac{d}{dx}\left[\sin\left(x^3+3x\right)\right]\right)}_{\text{derivative of inner}}$$

$$= 5\left(\sin\left(x^3+3x\right)\right)^4 \cdot \underbrace{\left[\cos\left(x^3+3x\right)\cdot\left(3x^2+3\right)\right]}_{\text{Chain Rule}}$$

i.e., $\frac{d}{dx}\left[\sin^5\left(x^3+3x\right)\right] = 5\left(\sin\left(x^3+3x\right)\right)^4 \cdot \cos\left(x^3+3x\right) \cdot \left(3x^2+3\right)$

10. Given that $x^2 - x^3 y^5 = \tan(y)$, compute $\frac{dy}{dx}$

i. Differentiate both sides w.r.t. $x$

$$\frac{d}{dx}\left[x^2 - \underbrace{x^3}_{1^{\text{st}}}\underbrace{y^5}_{2^{\text{nd}}}\right] = \frac{d}{dx}\left[\tan(y)\right]$$

$$\Rightarrow 2x - \left(\underbrace{3x^2}_{1^{\text{st}}\text{ prime}} \cdot \underbrace{y^5}_{2^{\text{nd}}} + \underbrace{5y^4\frac{dy}{dx}}_{2^{\text{nd}}\text{ prime}} \cdot \underbrace{x^3}_{1^{\text{st}}}\right) = \sec^2(y) \cdot \frac{dy}{dx}$$

Simplifying, we have:

$$2x - 3x^2y^5 - 5x^3y^4\frac{dy}{dx} = \sec^2(y)\frac{dy}{dx}$$

ii. Solve algebraically for $\frac{dy}{dx}$

a. Get $\frac{dy}{dx}$ terms on left side, all other terms on right side

$$\Rightarrow -5x^3y^4\frac{dy}{dx} - \sec^2(y)\frac{dy}{dx} = -2x + 3x^2y^5$$

b. Factor out $\frac{dy}{dx}$

$$\Rightarrow \frac{dy}{dx}\left(-5x^3y^4 - \sec^2(y)\right) = -2x + 3x^2y^5$$

c. Divide both sides by the cofactor of $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-2x+3x^2y^5}{-5x^3y^4-\sec^2(y)} = \frac{2x-3x^2y^5}{5x^3y^4+\sec^2(y)}$$

$$\boxed{\frac{dy}{dx} = \frac{-2x+3x^2y^5}{-5x^3y^4-\sec^2(y)} = \frac{2x-3x^2y^5}{5x^3y^4+\sec^2(y)}}$$

11. Given that $f(x) = 4x^2 - 5x + 5$, compute $f'(x)$ **using the definition of derivative.** (i.e., using the "limit process.")

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[4(x+\Delta x)^2 - 5(x+\Delta x)+5\right] - \left[4x^2-5x+5\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[4\left(x^2+2x\Delta x+\Delta x^2\right) - 5(x+\Delta x)+5\right] - \left[4x^2-5x+5\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[4x^2+8x\Delta x+4\Delta x^2-5x-5\Delta x+5\right] - \left[4x^2-5x+5\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{8x\Delta x+4\Delta x^2-5\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(8x+4\Delta x-5)}{\Delta x} = \lim_{\Delta x \to 0} (8x + 4\Delta x - 5) = 8x + 4(0) - 5 = 8x - 5$$

i.e., $f'(x) = 8x - 5$

**Extra** (Wow! 10 Points)

Given that $S'(x) = \frac{1}{2\sqrt{x}}$ (i.e., $\frac{d}{dx}[S(x)] = \frac{1}{2\sqrt{x}}$); compute $\frac{d}{dx}\left[S\left(x^2+2\right)\right]$

| | | |
|---|---|---|
| Outer: | $=$ | $S(\quad)$ |
| Deriv. of outer | $=$ | $\frac{1}{2\sqrt{\phantom{x}}}$ |

$$\frac{d}{dx}\Big[\underset{\substack{\uparrow\\ \text{outer}}}{S}\Big(\underset{\substack{\uparrow\\ \text{inner}}}{\underbrace{x^2+2}}\Big)\Big] = \underbrace{\frac{1}{2\sqrt{x^2+2}}}_{\text{Deriv of outer, eval. at inner}} \cdot \underbrace{2x}_{\text{deriv. of inner}} = \frac{2x}{2\sqrt{x^2+2}} = \frac{x}{\sqrt{x^2+2}}$$

i.e., $\frac{d}{dx}\left[S\left(x^2+2\right)\right] = \frac{2x}{2\sqrt{x^2+2}} = \frac{x}{\sqrt{x^2+2}}$