MTH 1125 Test #1 - (9 am class)

Fall 2016

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 2}\frac{x^2+4x-12}{x^2-8x+12} =$

Step #1 Try Plugging in:

$\lim_{x\to 2}\frac{x^2+4x-12}{x^2-8x+12} = \frac{(2)^2+4(2)-12}{(2)^2-8(2)+12} = \frac{0}{0}$ No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$\lim_{x\to 2}\frac{x^2+4x-12}{x^2-8x+12} = \lim_{x\to 3}\frac{(x+6)(x-2)}{(x-2)(x-6)} = \lim_{x\to 3}\frac{(x+6)}{(x-6)} = \frac{(2)+6}{(2)-6} = \frac{8}{-4} = -2$

i.e., $\lim_{x\to 2}\frac{x^2+4x-12}{x^2-8x+12} = -2$

2. Compute: $\lim_{x\to 1}\frac{x^2-5x+6}{x^2+7x-30} =$

Step #1 Try Plugging In:

$\lim_{x\to 1}\frac{x^2-5x+6}{x^2+7x-30} = \frac{(1)^2-5(1)+6}{(1)^2+7(1)-30} = -\frac{1}{11}$

i.e., $\lim_{x\to 1}\frac{x^2-5x+6}{x^2+7x-30} = -\frac{1}{11}$

3. Compute: $\lim_{x\to\ 3} \frac{x^2-4x-12}{x^2-8x+15} =$

Step #1 Try Plugging in:

$$\lim_{x\to\ 3} \frac{x^2-4x-12}{x^2-8x+15} = \frac{(3)^2-4(3)-12}{(3)^2-8(3)+15} = \frac{-15}{0}$$

No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

No Good! "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to\ 3^-} \frac{x^2-4x-12}{x^2-8x+15} = \lim_{x\to\ 3^-} \frac{x^2-4x-12}{(x-3)(x-5)} = \frac{-15}{(-\varepsilon)(-2)} = \frac{\left(\frac{15}{2}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{ll} & x \to 3^- \\ \Rightarrow & x < 3 \\ \Rightarrow & x - 3 < 0 \end{array}$$

$$\lim_{x\to\ 3} \frac{x^2-4x-12}{x^2-8x+15} = \lim_{x\to\ 3^+} \frac{x^2-4x-12}{(x-3)(x-5)} = \frac{-15}{(+\varepsilon)(-2)} = \frac{\left(\frac{15}{2}\right)}{(+\varepsilon)} = +\infty$$

$$\begin{array}{ll} & x \to 3^+ \\ \Rightarrow & x > 3 \\ \Rightarrow & x - 3 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to\ 3} \frac{x^2-4x-12}{x^2-8x+15}$ **Does Not Exist!**

4. Compute: $\lim_{x\to\ -\infty} \frac{x^3+3x^2-8x}{9x^3+4x-5} =$

$$\lim_{x\to\ -\infty} \frac{x^3+3x^2-8x}{9x^3+4x-5} = \lim_{x\to\ -\infty} \frac{x^3}{9x^3} = \lim_{x\to\ -\infty} \frac{1}{9} = \frac{1}{9}$$

i.e., $\lim_{x\to\ -\infty} \frac{x^3+3x^2-8x}{9x^3+4x-5} = \frac{1}{9}$

5. $f(x) = \frac{x^2-x-6}{x^2+x-6}$ Find the asymptotes and graph

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow x^2 + x - 6 = 0$

$\Rightarrow (x+3)(x-2) = 0$

$\Rightarrow x = -3$ and $x = 2$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$\lim_{x\to-3^-} \frac{x^2-x-6}{x^2+x-6} = \lim_{x\to-3^-} \frac{x^2-x-6}{(x+3)(x-2)} = \frac{6}{(-\varepsilon)(-5)} = \frac{\left(-\frac{6}{5}\right)}{-\varepsilon} = +\infty$

$$\begin{array}{ll} & x \to -3^- \\ \Rightarrow & x < -3 \\ \Rightarrow & x+3 < 0 \end{array}$$

$\lim_{x\to-3^+} \frac{x^2-x-6}{x^2+x-6} = \lim_{x\to-3^+} \frac{x^2-x-6}{(x+3)(x-2)} = \frac{6}{(\varepsilon)(-5)} = \frac{\left(-\frac{6}{5}\right)}{\varepsilon} = -\infty$

$$\begin{array}{ll} & x \to -3^+ \\ \Rightarrow & x > -3 \\ \Rightarrow & x+3 > 0 \end{array}$$

Since the one-sided limits are infinite, $x = -3$ *is a vertical asymptote.*

$\lim_{x\to2^-} \frac{x^2-x-6}{x^2+x-6} = \lim_{x\to2^-} \frac{x^2-x-6}{(x+3)(x-2)} = \frac{-4}{(5)(-\varepsilon)} = \frac{\left(\frac{4}{5}\right)}{(\varepsilon)} = +\infty$

$$\begin{array}{ll} & x \to 2^- \\ \Rightarrow & x < 2 \\ \Rightarrow & x-2 < 0 \end{array}$$

$\lim_{x\to2^+} \frac{x^2-x-6}{x^2+x-6} = \lim_{x\to2^+} \frac{x^2-x-6}{(x+3)(x-2)} = \frac{-4}{(5)(\varepsilon)} = \frac{\left(-\frac{4}{5}\right)}{(\varepsilon)} = -\infty$

$$\begin{array}{ll} & x \to 2^+ \\ \Rightarrow & x > 2 \\ \Rightarrow & x-2 > 0 \end{array}$$

Since the one-sided limits are **infinite,** $x = 2$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{x^2-x-6}{x^2+x-6} = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1$

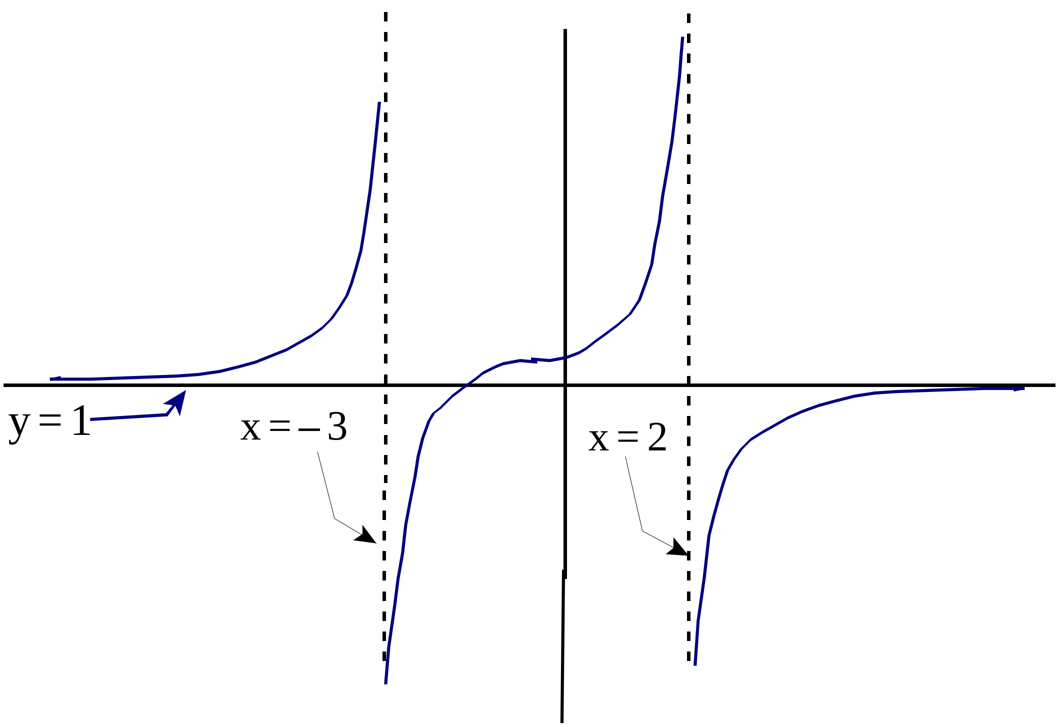
$\lim_{x\to+\infty} \frac{x^2-x-6}{x^2+x-6} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are **finite** and **constant,** $y = 1$ *is a horizontal asymptotes.*

Summary:

$\lim_{x\to-3^-} \frac{x^2-x-6}{x^2+x-6} = +\infty$	
$\lim_{x\to-3^+} \frac{x^2-x-6}{x^2+x-6} = -\infty$	$\lim_{x\to-\infty} \frac{x^2-x-6}{x^2+x-6} = 1$
$\lim_{x\to2^-} \frac{x^2-x-6}{x^2+x-6} = +\infty$	$\lim_{x\to+\infty} \frac{x^2-x-6}{x^2+x-6} = 1$
$\lim_{x\to2^+} \frac{x^2-x-6}{x^2+x-6} = -\infty$	

Graph $f(x) = \frac{x^2-x-6}{x^2+x-6}$



6. Compute: $\lim_{x\to 2} \frac{\sqrt{11-x}-3}{x-2} =$

Step #1 Try Plugging in:

$$\lim_{x\to 2} \frac{\sqrt{11-x}-3}{x-2} = \frac{\sqrt{11-(2)}-3}{(2)-2} = \frac{0}{0}$$

No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 2} \frac{\sqrt{11-x}-3}{x-2} = \lim_{x\to 2} \frac{\sqrt{11-x}-3}{x-2} \cdot \frac{\sqrt{11-x}+3}{\sqrt{11-x}+3} = \lim_{x\to 2} \frac{\left(\sqrt{11-x}\right)^2-(3)^2}{(x-2)\left[\sqrt{11-x}+3\right]}$$

$$= \lim_{x\to 2} \frac{(11-x)-9}{(x-2)\left[\sqrt{11-x}+3\right]} = \lim_{x\to 2} \frac{(2-x)}{(x-2)\left[\sqrt{11-x}+3\right]} = \lim_{x\to 2} \frac{-(x-2)}{(x-2)\left[\sqrt{11-x}+3\right]}$$

$$= \lim_{x\to 2} \frac{-1}{\left[\sqrt{11-x}+3\right]} = \lim_{x\to 2} \frac{-1}{\left[\sqrt{11-(2)}+3\right]} = \frac{-1}{[3+3]} = -\frac{1}{6}$$

i.e., $\lim_{x\to 2} \frac{\sqrt{11-x}-3}{x-2} = -\frac{1}{6}$

7.

$x =$	$f(x) =$
-2.5	-9.1
-2.1	-90.8
-2.01	-900.3
-2.001	$-9,000.3$
-2.0001	$-90,000.9$

$x =$	$f(x) =$
-1.5	9.1
-1.9	90.8
-1.99	900.3
-1.999	$9,000.3$
-1.9999	$90,000.9$

Based on the information in the table above, do the following:

(a) $\lim_{x \to -2^-} f(x) = -\infty$

(b) $\lim_{x \to -2^+} f(x) = +\infty$

(c) Graph $f(x)$