MTH 4441 Homework #8 - Permutations - Solutions

Fall 2023

Pat Rossi Name ______________________

For exercises **??-??**:

[1] Express the given permutation on the set $\{1, 2, 3, \ldots, n\}$ into the "product" of disjoint cycles

[2] Express the given permutation on the set $\{1, 2, 3, \ldots, n\}$ into the "product" of transpositions

[3] Classify the permutation as being either "even" or "odd"

1. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 1 & 4 & 2 & 5 \end{pmatrix} = (1,3) \circ (2,6,5) = (1,3) \circ \underbrace{(2,5) \circ (2,6)}_{(2,6,5)}$

   Alternatively:

   $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 1 & 4 & 2 & 5 \end{pmatrix} = (2,6,5) \circ (1,3) = \underbrace{(2,5) \circ (2,6)}_{(2,6,5)} \circ (1,3)$

   Since $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 1 & 4 & 2 & 5 \end{pmatrix}$ can be written as the product of 3 transpositions, it is an **odd permutation.**

2. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 1 & 7 & 4 & 8 & 5 & 2 \end{pmatrix} = (1,3) \circ (2,6,8) \circ (4,7,5)$

   $= (1,3) \circ \underbrace{(2,8) \circ (2,6)}_{(2,6,8)} \circ \underbrace{(4,5) \circ (4,7)}_{(4,7,5)}$

   (Other Answers Are Possible)

   Since $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 1 & 7 & 4 & 8 & 5 & 2 \end{pmatrix}$ can be written as the product of 5 transpositions, it is an **odd permutation.**

3. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 7 & 5 & 1 & 3 & 6 & 8 & 4 \end{pmatrix} = (1,2,7,8,4) \circ (3,5)$

$$= \underbrace{(1,4) \circ (1,8) \circ (1,7) \circ (1,2)}_{(1,2,7,8,4)} \circ (3,5)$$

(Other Answers Are Possible)

Since $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 7 & 5 & 1 & 3 & 6 & 8 & 4 \end{pmatrix}$ can be written as the product of 5 transpositions,

it is an **odd permutation.**

4. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 8 & 7 & 6 & 3 & 2 & 5 & 1 \end{pmatrix} = (1,4,6,2,8) \circ (3,7,5)$

$$= \underbrace{(1,8) \circ (1,2) \circ (1,6) \circ (1,4)}_{(1,4,6,2,8)} \circ \underbrace{(3,5) \circ (3,7)}_{(3,7,5)}$$

(Other Answers Are Possible)

Since $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 8 & 7 & 6 & 3 & 2 & 5 & 1 \end{pmatrix}$ can be written as the product of 6 transpositions,

it is an **even permutation.**

5. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 7 & 1 & 3 & 8 & 6 & 2 & 5 \end{pmatrix} = (1,4,3) \circ (2,7) \circ (5,8)$

$$= \underbrace{(1,3) \circ (1,4)}_{(1,4,3)} \circ (2,7) \circ (5,8)$$

(Other Answers Are Possible)

Since $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 7 & 1 & 3 & 8 & 6 & 2 & 5 \end{pmatrix}$ can be written as the product of 4 transpositions,

it is an **even permutation.**

6. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 2 & 3 & 1 & 8 & 4 & 7 \end{pmatrix} = (1,5) \circ (2,6,8,7,4,3)$

$$= (1,5) \circ \underbrace{(2,3) \circ (2,4) \circ (2,7) \circ (2,8) \circ (2,6)}_{(2,6,8,7,4,3)}$$

(Other Answers Are Possible)

Since $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 2 & 3 & 1 & 8 & 4 & 7 \end{pmatrix}$ can be written as the product of 6 transpositions, it is an **even permutation.**