MTH 3318 - Test #2

Fall 2014 - Solutions

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Instructions. Fully document your work.

1. In exercises 1.a - 1.d, let p be the statement: "We have plenty of rain," and let q be the statement: "our flowers will grow." Write each statement in symbolic form.

   (a) $\text{If } \underbrace{\text{we have plenty of rain,}}_{p} \underbrace{\text{then}}_{\rightarrow} \underbrace{\text{our flowers will grow.}}_{q}$

   $p \rightarrow q$

   (b) $\underbrace{\text{We will have plenty of rain,}}_{p} \underbrace{\text{or}}_{\vee} \underbrace{\text{our flowers will not grow.}}_{\sim q}$

   $p \vee \sim q$

   (c) $\underbrace{\text{Our having plenty of rain}}_{p} \underbrace{\text{is a necessary and sufficient condition for}}_{\leftrightarrow} \underbrace{\text{our flowers to grow.}}_{q}$

   $p \leftrightarrow q$

   (d) $\underbrace{\text{We will have plenty of rain}}_{p} \underbrace{\text{if}}_{\leftarrow} \underbrace{\text{our flowers grow.}}_{q}$

   $p \leftarrow q$ or $q \rightarrow p$

2. In exercises 2.a - 2.d, let p be the statement: "$f(x)$ is continuous" and let q be the statement: "$f(x)$ is differentiable." Write each statement in words.

   (a) $p \wedge q$

   $\underbrace{f(x) \text{ is continuous}}_{p} \underbrace{\text{and}}_{\wedge} \underbrace{f(x) \text{ is differentiable.}}_{q}$

   (b) $p \vee q$

   $\underbrace{f(x) \text{ is continuous}}_{p} \underbrace{\text{or}}_{\vee} \underbrace{f(x) \text{ is differentiable.}}_{q}$

   (c) $q \rightarrow \sim p$

   $\text{If } \underbrace{f(x) \text{ is differentiable,}}_{q} \underbrace{\text{then}}_{\rightarrow} \underbrace{f(x) \text{ is not continuous.}}_{\sim p}$

(d) $\sim p \quad \leftrightarrow \sim q$

$\underbrace{f(x) \text{ is not continuous}}_{\sim p} \underbrace{\text{if and only if}}_{\leftrightarrow} \underbrace{f(x) \text{ is not differentiable.}}_{\sim q}$

3. In problems 3.a - 3.d, determine whether the given propositions are True or False:

(a) If $\underbrace{3^2 = 9}_{T}$, then $\underbrace{8 > 10}_{F}$.

$T \rightarrow F \equiv F$

The proposition is FALSE.

(b) If $\underbrace{3^2 > 3}_{T}$, then $\underbrace{3^2 > 5}_{T}$.

$T \rightarrow T = T$

The proposition is TRUE.

(c) If $\underbrace{3^2 > 10}_{F}$ if and only if $\underbrace{3^2 = 5}_{F}$.

$F \leftrightarrow F = T$

The proposition is TRUE.

(d) If $\underbrace{3^2 = 5}_{F}$, then $\underbrace{3^2 > 10}_{F}$.

$F \rightarrow F = T$

The proposition is TRUE.

4. In exercises 4.a-4.b construct a truth table for the statement given.

(a) $p \vee (q \longleftrightarrow r)$

p	q	r	$(q \longleftrightarrow r)$	$p \vee (q \longleftrightarrow r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	T	T

(b) $(\sim p \longrightarrow q) \wedge \sim r$

p	q	r	$\sim p$	$\sim r$	$(\sim p \longrightarrow q)$	$(\sim p \longrightarrow q) \wedge \sim r$
T	T	T	F	F	T	F
T	T	F	F	T	T	T
T	F	T	F	F	T	F
T	F	F	F	T	T	T
F	T	T	T	F	T	F
F	T	F	T	T	T	T
F	F	T	T	F	F	F
F	F	F	T	T	F	F

5. For problems 5.a - 5.d, negate the given statements:

   (a) All toads have warts.

   Possible negations:

   Some toads don't have warts.

   At least one toad doesn't have warts.

   There exists a toad that doesn't have warts.

   (b) Some birds can fly.

   Possible negations:

   No birds can fly.

   There does not exist a bird can fly.

   (c) Some submarines do not have screen doors.

   Possible negations:

   All submarines have screen doors.

   (d) $\forall$ real numbers x, $\exists$ real number y, $\ni x + y = x$.
   (i.e. For all real numbers x, there exists a real number y such that $x + y = x$.)

   $\sim$ ($\forall$ real numbers x, $\exists$ real number y, $\ni x + y = x$.)

   $\equiv \exists$ a real number x, $\ni \sim$ ($\exists$ a real number y, $\ni$ $x + y = x$.)

   $\equiv \exists$ a real number x, $\ni \forall$ real numbers y, $\sim (x + y = x.)$

   $\equiv \exists$ a real number x, $\ni \forall$ real numbers y, $x + y \neq x$.

$\exists$ a real number x, $\ni \forall$ real numbers y, $x + y \neq x$.

6. For problems 6.a - 6.b, disprove the given statements by providing a suitable counter-example:

   (a) $\forall n \in \mathbb{N}$, if $2n + 1$ is prime, then n is even.

   Counter-example:

   Let $n = 1$.

   Then $2n + 1 = 3$ is prime, but n is odd.

   Hence our statement if false by counter-example.

   (b) For all integers x, y, and z, if x is a factor of $(y + z)$, then x is a factor of y and x is a factor of z.

   Counter-example:

   Let $x = 2$, $y = 3$, and $z = 5$.

   Then x is a factor of $(y + z)$, but x is neither a factor of y or z.

7. Write the converse, inverse, and contrapositive of the following statement, labeling each one.

If I eat my vegetables, then I will grow "big and strong."

$$\text{If } \underbrace{\text{I eat my vegetables,}}_{p}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{\text{I will grow ``big and strong.''}}_{q}$$

converse:

$$\text{If } \underbrace{\text{I grow ``big and strong.''}}_{q}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{\text{I will eat my vegetables.}}_{p}$$

inverse:

$$\text{If } \underbrace{\text{I do not eat my vegetables,}}_{\sim p}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{\text{I will not grow ``big and strong.''}}_{\sim q}$$

contrapositive:

$$\text{If } \underbrace{\text{I do not grow ``big and strong.''}}_{\sim q}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{\text{I will not eat my vegetables.}}_{\sim p}$$

8. In problems 8.a - 8.b, determine whether the given arguments are valid.

(a) I will go to the concert if and only if I finish studying. If I get to bed late, then I will have finished studying. Therefore, I will have been to the concert if I go to bed late.

If we make the following assignments:

p : I will go to the concert.

r : I finish studying.

s : I get to bed late.

Then our argument has the form:

$$p_1 : \underbrace{\text{I will go to the concert}}_{p}\ \underbrace{\text{if and only if}}_{\leftrightarrow}\ \underbrace{\text{I finish studying.}}_{r}$$

$$p_2 : \text{If } \underbrace{\text{I get to bed late,}}_{s}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{\text{I will have finished studying.}}_{r}$$

$$q : \underbrace{\text{Therefore,}}_{\therefore}\ \underbrace{\text{I will have been to the concert}}_{p}\ \underbrace{\text{ if }}_{\leftarrow}\ \underbrace{\text{I go to bed late.}}_{s}$$

i.e., Our argument has the form: $(p_1 \wedge p_2) \rightarrow q$

p	r	s	$p_1 : (p \leftrightarrow r)$	$p_2 : (s \rightarrow r)$	$(p_1 \wedge p_2)$	$q : (s \rightarrow p)$	$(p_1 \wedge p_2) \rightarrow q$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	T
T	F	F	F	T	F	T	T
F	T	T	F	T	F	F	T
F	T	F	F	T	F	T	T
F	F	T	T	F	F	F	T
F	F	F	T	T	T	T	T

Note the argument is a tautology. Therefore, it is VALID.

(b) Some desks are made of wood. All paper is made of wood. Therefore, some desks are made of paper.

If we make the following assignments:

$D-$ Desks

$W-$ Things made of Wood

$P-$ Paper and things that are made of Paper

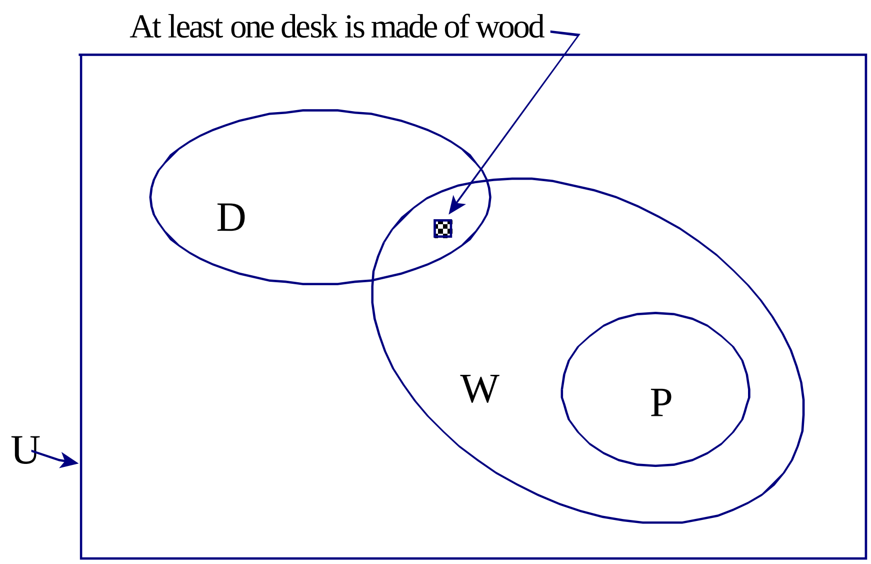
Then our argument has the form:

p_1 : Some desks are made of wood.

p_2 : All paper is made of wood.

$\therefore q$: Therefore, some desks are made of paper.

The argument is shown below, using Euler Circles.



Since the Conclusion can be drawn in such a way that it is False, while the Premises are drawn so as to make them true, the argument is INVALID.

9. In problems 9.a - 9.b, determine whether the given arguments are valid.

(a) If I'm thrifty and I save my money, then I will buy a bicycle. I will buy a bicycle. Therefore, if I don't buy a bicycle, then I will not have been thrifty.

If we make the following assignments:

p : I'm thrifty.

r : I save my money.

s : I will buy a bicycle.

Then our argument has the form:

p_1 : If $\underbrace{\text{I'm thrifty}}_{(p}$ $\underbrace{\text{and}}_{\wedge}$ $\underbrace{\text{I save my money,}}_{r)}$ $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{\text{I will buy a bicycle.}}_{s}$

p_2 : $\underbrace{\text{I will buy a bicycle.}}_{s}$

q : $\underbrace{\text{Therefore,}}_{\therefore}$ if $\underbrace{\text{I don't buy a bicycle,}}_{\sim s}$ $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{\text{I will not have been thrifty.}}_{\sim p}$

i.e., Our argument has the form: $(p_1 \wedge p_2) \rightarrow q$

p	r	s	$\sim p$	$\sim s$	$p \wedge r$	$p_1 : (p \wedge r) \rightarrow s$	$p_2 : s$	$(p_1 \wedge p_2)$	$q : \sim s \rightarrow \sim p$	$(p_1 \wedge p_2) \rightarrow q$
T	T	T	F	F	T	T	T	T	T	T
T	T	F	F	T	T	F	F	F	F	T
T	F	T	F	F	F	T	T	T	T	T
T	F	F	F	T	F	T	F	F	F	T
F	T	T	T	F	F	T	T	T	T	T
F	T	F	T	T	F	T	F	F	T	T
F	F	T	T	F	F	T	T	T	T	T
F	F	F	T	T	F	T	F	F	T	T

Note the argument is tautology. Therefore, it is VALID.

(b) No ducks are birds. Some animals are birds. Therefore, no ducks are animals.

If we make the following assignments:

$D-$ Ducks

$B-$ Birds

$A-$ Animals

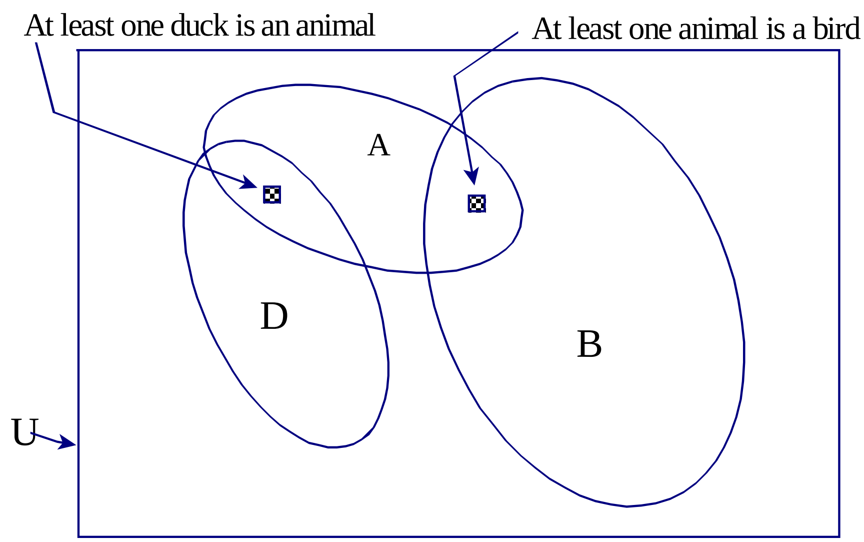
Then our argument has the form:

p_1 : No ducks are birds.

p_2 : Some animals are birds.

$\therefore q$: Therefore, no ducks are animals.

The argument is shown below, using Euler Circles.



Since the Conclusion can be drawn in such a way that it is False, while the Premises are drawn so as to make them true, the argument is INVALID.