MTH 1125 2pm Class - Test #4- Solutions

Fall 2022

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Show CLEARLY how you arrive at your answers!

1. **Compute:** $\int (28x^3 + 18x^2 - 10x + 6\sqrt{x} + 3)\, dx$

$= \int \left(28x^3 + 18x^2 - 10x + 6x^{\frac{1}{2}} + 3\right) dx = 28\left[\frac{x^4}{4}\right] + 18\left[\frac{x^3}{3}\right] - 10\left[\frac{x^2}{2}\right] + 6\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right] + 3x + C$

$= 7x^3 + 6x^2 - 5x + 6\left[\left(\frac{2}{3}\right) x^{\frac{3}{2}}\right] + 3x + C = 7x^3 + 6x^2 - 5x + 4x^{\frac{3}{2}} + 3x + C$

i.e., $\int (28x^3 + 18x^2 - 10x + 6\sqrt{x} + 3)\, dx = 7x^3 + 6x^2 - 5x + 4x^{\frac{3}{2}} + 3x + C$

2. **Compute:** $\int \left(6x^2+18x+8\right)^4 (2x+3)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(6x^2+18x+8\right)^4$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = (6x^2+18x+8)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{(6x^2+18x+8)}_{\text{function}} - - - - \rightarrow \underbrace{(2x+3)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = (6x^2+18x+8)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 6x^2+18x+8 \\ \Rightarrow \frac{du}{dx} &= 12x+18 \\ \Rightarrow du &= (12x+18)\,dx \\ \Rightarrow \frac{1}{6}du &= (2x+3)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\left(6x^2+18x+8\right)^4}_{u^4} \underbrace{(2x+3)\,dx}_{\frac{1}{6}du} = \int u^4 \tfrac{1}{6}du = \tfrac{1}{6}\int u^4 du$$

4. Integrate (in terms of u).

$$\tfrac{1}{6}\int u^4 du = \tfrac{1}{6}\left[\tfrac{u^5}{5}\right] + C = \tfrac{1}{30}u^5 + C$$

5. Re-express in terms of the original variable, x.

$$\int \left(6x^2+18x+8\right)^4 (2x+3)\,dx = \underbrace{\frac{1}{30}\left(6x^2+18x+8\right)^5 + C}_{\frac{1}{30}u^5}$$

$$\text{i.e., } \int \left(6x^2+18x+8\right)^4 (2x+3)\,dx = \frac{1}{30}\left(6x^2+18x+8\right)^5 + C$$

3. **Compute:** $\int \left(2\sin(x) - 5\csc^2(x) + 4\sec(x)\tan(x)\right) dx =$

$\int \left(2\sin(x) - 5\csc^2(x) + 4\sec(x)\tan(x)\right) dx = 2\left(-\cos(x)\right) - 5\left(-\cot(x)\right) + 4\sec(x) + C$

$= -2\cos(x) + 5\cot(x) + 4\sec(x) + C$

$$\text{i.e., } \int \left(2\sin(x) - 5\csc^2(x) + 4\sec(x)\tan(x)\right) dx = -2\cos(x) + 5\cot(x) + 4\sec(x) + C$$

4. **Compute:** $\int \sin\left(4x^3+9x+8\right)\left(4x^2+3\right)dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\sin\left(4x^3+9x+8\right)$

   $\nearrow$ $\nwarrow$

   outer inner

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(4x^3+9x+8\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(4x^3+9x+8\right)}_{\text{function}} ----\rightarrow \underbrace{\left(4x^2+3\right)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(4x^3+6x+3\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 4x^3+9x+8 \\ \Rightarrow \frac{du}{dx} &= 12x^2+9 \\ \Rightarrow du &= \left(12x^2+9\right)dx \\ \Rightarrow \frac{1}{3}du &= \left(4x^2+3\right)dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\sin\left(4x^3+9x+8\right)}_{\sin(u)}\underbrace{\left(4x^2+3\right)dx}_{\frac{1}{3}du} = \int \sin(u)\ \frac{1}{3}du = \frac{1}{3}\int \sin(u)\,du$$

4. Integrate (in terms of u).

$$\frac{1}{3}\int \sin(u)\,du = \frac{1}{3}\left[-\cos(u)\right]+C = -\frac{1}{3}\cos(u)+C$$

5. Re-express in terms of the original variable x.

$$\int \sin\left(4x^3+9x+8\right)\left(4x^2+3\right)dx = \underbrace{-\frac{1}{3}\cos\left(4x^3+9x+8\right)+C}_{-\frac{1}{3}\cos(u)+C}$$

$$\text{i.e., } \int \sin\left(4x^3+9x+8\right)\left(4x^2+3\right)dx = -\tfrac{1}{3}\cos\left(4x^3+9x+8\right)+C$$

5. **Compute:** $\int_{-1}^{1}\left(6x^3+9x^2+2\right)dx =$

$$\int_{-1}^{1}\underbrace{\left(6x^3+9x^2+2\right)}_{f(x)}dx = \underbrace{\left[6\frac{x^4}{4}+9\frac{x^3}{3}+2x\right]_{-1}^{1}}_{F(x)} = \underbrace{\left[\frac{3}{2}x^4+3x^3+2x\right]_{-1}^{1}}$$

$$= \underbrace{\left[\frac{3}{2}(1)^4+3(1)^3+2(1)\right]}_{F(1)} - \underbrace{\left[\frac{3}{2}(-1)^4+3(-1)^3+2(-1)\right]}_{F(-1)}$$

$$= \tfrac{13}{2} - \left(-\tfrac{7}{2}\right) = 10$$

$$\text{i.e., } \int_{-1}^{1}\left(6x^3+9x^2+2\right)dx = 10$$

6. **Compute:** $\int_{-1}^{1} (x^3+x+1)^3 (3x^2+1)\ dx =$ (The answer may not be a whole number)

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $(x^3+x+1)^3$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = (x^3+x+1)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{(x^3+x+1)}_{\text{function}} - - - - \rightarrow \underbrace{(3x^2+1)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = (x^3+x+1)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= x^3+x+1 \\ \Rightarrow \tfrac{du}{dx} &= 3x^2+1 \\ \Rightarrow du &= (3x^2+1)\ dx \\ &= \end{aligned}$$

When $x=-1$, $u = x^3+x+1 = (-1)^3 + (-1) + 1 = -1$
When $x=1$, $u = x^3+x+1 = (1)^3 + (1) + 1 = 3$

3. Analyze in terms of u and du

$$\int_{x=-1}^{x=1} \underbrace{(x^3+x+1)^3}_{u^3} \underbrace{(3x^2+1)\ dx}_{du} = \int_{u=-1}^{u=3} u^3 du =$$

Don't forget to re-write the limits of integration in terms of u!

4. Integrate (in terms of u).

$$\int_{u=-1}^{u=3} u^3 du = \left[\frac{u^4}{4}\right]_{u=-1}^{u=3} = \underbrace{\frac{(3)^4}{4}}_{F(3)} - \underbrace{\frac{(-1)^4}{4}}_{F(-1)} = \frac{81}{4} - \frac{1}{4} = \frac{80}{4} = 20$$

i.e., $\int_{-1}^{1} \left(x^3 + x + 1\right)^3 \left(3x^2 + 1\right)\, dx = 20$

7. **Compute:** $\frac{d}{dx}\left[\ln\left(2x^5 + 5x^2 - 10x\right)\right] =$

$$\underbrace{\frac{d}{dx}\left[\ln\left(2x^5 + 5x^2 - 10x\right)\right]}_{\frac{d}{dx}[\ln(g(x))]} = \underbrace{\frac{1}{2x^5 + 5x^2 - 10x}}_{\frac{1}{g(x)}} \cdot \underbrace{\left(10x^4 + 10x - 10\right)}_{g'(x)} = \frac{10x^4+10x-10}{2x^5+5x^2-10x}$$

i.e., $\frac{d}{dx}\left[\ln\left(2x^5 + 5x^2 - 10x\right)\right] = \frac{10x^4+10x-10}{2x^5+5x^2-10x}$

Extra! (Wow! - 5 pts. Can you believe it?) **Compute:** $\frac{d}{dx}\left[\ln\left(\sqrt{\frac{3x^4-2x^2+1}{x^3-x}}\right)\right] =$

$$\frac{d}{dx}\left[\ln\left(\sqrt{\frac{3x^4-2x^2+1}{x^3-x}}\right)\right] \underset{\text{rewrite}}{=} \frac{d}{dx}\left[\ln\left[\left(\frac{3x^4-2x^2+1}{x^3-x}\right)^{\frac{1}{2}}\right]\right] \underset{\text{rewrite}}{=} \frac{d}{dx}\left[\frac{1}{2}\ln\left(\frac{3x^4-2x^2+1}{x^3-x}\right)\right]$$

$$\underset{\text{rewrite}}{=} \frac{1}{2}\frac{d}{dx}\left[\ln\left(\frac{3x^4-2x^2+1}{x^3-x}\right)\right] \underset{\text{rewrite}}{=} \frac{1}{2}\frac{d}{dx}\left[\ln\left(3x^4-2x^2+1\right)-\ln\left(x^3-x\right)\right]$$

$$\underset{\text{rewrite}}{=} \frac{1}{2}\left(\frac{1}{3x^4-2x^2+1}\left(12x^3-4x\right)-\frac{1}{x^3-x}\left(3x^2-1\right)\right) \underset{\text{rewrite}}{=} \frac{6x^3-2x}{3x^4-2x^2+1}-\frac{3x^2-1}{2x^3-2x}$$

$$\boxed{\frac{d}{dx}\left[\ln\left(\sqrt{\frac{3x^4-2x^2+1}{x^3-x}}\right)\right] = \frac{6x^3-2x}{3x^4-2x^2+1}-\frac{3x^2-1}{2x^3-2x}}$$