1. $\frac{3x^3+2x^2-4x+1}{x^2+1} = 3x + 2 + \frac{-7x-1}{x^2+1} = 3x + 2 - \frac{7x+1}{x^2+1}$

Here's how we arrived at the result:

Compare the leading term of the divisor $(x^2 + 1)$ with the leading term of the dividend $(3x^3 + 2x^2 - 4x + 1)$

x^2 is the leading term of the divisor

$3x^3$ is the leading term of the dividend

x^2 "goes into" $3x^3$ $3x$ times.

$$
\begin{array}{ccc|ccccccc}
 & & & & & & & 3x & & \\
\cline{4-10}
x^2 & + & 1 & 3x^3 & + & 2x^2 & - & 4x & + & 1 \\
 & & & 3x^3 & & & + & 3x & & \\
\cline{4-10}
 & & & & & 2x^2 & - & 7x & + & 1
\end{array}
$$

Compare the leading term of the divisor $(x^2 + 1)$ with the leading term of the remainder $(2x^2 - 7x + 1)$

x^2 is the leading term of the divisor

$2x^2$ is the leading term of the remainder

x^2 "goes into" $2x^2$ 2 times.

$$
\begin{array}{ccc|ccccccc}
 & & & & & & & 3x & + & 2 \\
\cline{4-10}
x^2 & + & 1 & 3x^3 & + & 2x^2 & - & 4x & + & 1 \\
 & & & 3x^3 & & & + & 3x & & \\
\cline{4-10}
 & & & & & 2x^2 & - & 7x & + & 1 \\
 & & & & & 2x^2 & & & + & 2 \\
\cline{6-10}
 & & & & & & - & 7x & - & 1
\end{array}
$$

Compare the leading term of the divisor $(x^2 + 1)$ with the leading term of the remainder $-7x$, we find that the remainder is of lesser degree than the divisor

The process terminates with us dividing $-7x - 1$ by $x^2 + 1$.

$$
\begin{array}{ccc|ccccccccc}
 & & & & & & & 3x & + & 2 & + & \frac{-7x-1}{x^2+1} \\
\cline{4-10}
x^2 & + & 1 & 3x^3 & + & 2x^2 & - & 4x & + & 1 & & \\
 & & & 3x^3 & & & + & 3x & & & & \\
\cline{4-10}
 & & & & & 2x^2 & - & 7x & + & 1 & & \\
 & & & & & 2x^2 & & & + & 2 & & \\
\cline{6-10}
 & & & & & & - & 7x & - & 1 & & \\
 & & & & & & - & 7x & - & 1 & & \\
\cline{7-10}
 & & & & & & & & & 0 & &
\end{array}
$$

i.e., $\frac{3x^3+2x^2-4x+1}{x^2+1} = 3x + 2 + \frac{-7x-1}{x^2+1} = 3x + 2 - \frac{7x+1}{x^2+1}$

2. $\frac{3x^3+2x^2+1}{x^2+x+1} = 3x - 1 + \frac{-2x+2}{x^2+x+1} = 3x - 1 - \frac{2x-2}{x^2+x+1}$

Here's how we arrived at the result:

Compare the leading term of the divisor $(x^2 + x + 1)$ with the leading term of the dividend $(3x^3 + 2x^2 + 1)$

x^2 is the leading term of the divisor

$3x^3$ is the leading term of the dividend

x^2 "goes into" $3x^3$ $3x$ times.

$$\begin{array}{rrrrr|rrrrrrrr}
 & & & & & & & & & 3x & & \\
\cline{6-13}
x^2 & + & x & + & 1 & 3x^3 & + & 2x^2 & & & + & 1 \\
 & & & & & 3x^3 & + & 3x^2 & + & 3x & & \\
\cline{6-13}
 & & & & & & - & x^2 & - & 3x & + & 1
\end{array}$$

Compare the leading term of the divisor $(x^2 + x + 1)$ with the leading term of the remainder $(x^2 - 3x + 1)$

x^2 is the leading term of the divisor

$-x^2$ is the leading term of the remainder

x^2 "goes into" $-x^2$ -1 time.

$$\begin{array}{rrrrr|rrrrrrr}
 & & & & & & & & & 3x & - & 1 \\
\cline{6-12}
x^2 & + & x & + & 1 & 3x^3 & + & 2x^2 & & & + & 1 \\
 & & & & & 3x^3 & + & 3x^2 & + & 3x & & \\
\cline{6-12}
 & & & & & & - & x^2 & - & 3x & + & 1 \\
 & & & & & & - & x^2 & - & x & - & 1 \\
\cline{7-12}
 & & & & & & & & - & 2x & + & 2
\end{array}$$

Compare the leading term of the divisor $(x^2 + x + 1)$ with the leading term of the remainder $-2x$, we find that the remainder is of lesser degree than the divisor

The process terminates with us dividing $-2x + 2$ by $x^2 + x + 1$.

$$\begin{array}{rrrrr|rrrrrrrrr}
 & & & & & & & & & 3x & - & 1 & + & \frac{-2x+2}{x^2+x+1} \\
\cline{6-12}
x^2 & + & x & + & 1 & 3x^3 & + & 2x^2 & & & + & 1 & & \\
 & & & & & 3x^3 & + & 3x^2 & + & 3x & & & & \\
\cline{6-12}
 & & & & & & - & x^2 & - & 3x & + & 1 & & \\
 & & & & & & - & x^2 & - & x & - & 1 & & \\
\cline{7-12}
 & & & & & & & & - & 2x & + & 2 & & \\
 & & & & & & & & - & 2x & + & 2 & & \\
\cline{9-12}
 & & & & & & & & & & & 0 & &
\end{array}$$

i.e., $\frac{3x^3+2x^2+1}{x^2+x+1} = 3x - 1 + \frac{-2x+2}{x^2+x+1} = 3x - 1 - \frac{2x-2}{x^2+x+1}$

3. $\frac{2x^4+3x^2-2x+2}{(x-1)^2} = \frac{2x^4+3x^2-2x+2}{x^2-2x+1} = 2x^2+4x+9+\frac{12x-7}{x^2-2x+1}$

Here's how we arrived at the result:

Compare the leading term of the divisor (x^2-2x+1) with the leading term of the dividend $(2x^4+3x^2-2x+2)$

x^2 is the leading term of the divisor

$2x^4$ is the leading term of the dividend

x^2 "goes into" $2x^4$ $2x^2$ times.

$$
\begin{array}{r|l}
 & 2x^2 \\
\hline
x^2 - 2x + 1 & 2x^4 \qquad\qquad + 3x^2 - 2x + 2 \\
 & 2x^4 - 4x^3 + 2x^2 \\
\hline
 & \qquad 4x^3 + x^2 - 2x + 2
\end{array}
$$

Compare the leading term of the divisor (x^2-2x+1) with the leading term of the remainder $(4x^3+x^2-2x+2)$

x^2 is the leading term of the divisor

$4x^3$ is the leading term of the remainder

x^2 "goes into" $4x^3$ $4x$ times.

$$
\begin{array}{r|l}
 & 2x^2 + 4x \\
\hline
x^2 - 2x + 1 & 2x^4 \qquad\qquad + 3x^2 - 2x + 2 \\
 & 2x^4 - 4x^3 + 2x^2 \\
\hline
 & \qquad 4x^3 + x^2 - 2x + 2 \\
 & \qquad 4x^3 - 8x^2 + 4x \\
\hline
 & \qquad\qquad 9x^2 - 6x + 2
\end{array}
$$

Compare the leading term of the divisor (x^2-2x+1) with the leading term of the remainder $(9x^2-6x+2)$

x^2 is the leading term of the divisor

$9x^2$ is the leading term of the remainder

x^2 "goes into" $9x^2$ 9 times.

$$
\begin{array}{rrrrrrrrrrrrrr}
 & & & & & & & & 2x^2 & + & 4x & + & 9 \\
\cline{5-13}
x^2 & - & 2x & + & \multicolumn{1}{r|}{1} \; 2x^4 & & & + & 3x^2 & - & 2x & + & 2 \\
 & & & & 2x^4 & - & 4x^3 & + & 2x^2 & & & & \\
\cline{5-13}
 & & & & & & 4x^3 & + & x^2 & - & 2x & + & 2 \\
 & & & & & & 4x^3 & - & 8x^2 & + & 4x & & \\
\cline{7-13}
 & & & & & & & & 9x^2 & - & 6x & + & 2 \\
 & & & & & & & & 9x^2 & - & 18x & + & 9 \\
\cline{9-13}
 & & & & & & & & & & 12x & - & 7
\end{array}
$$

Compare the leading term of the divisor $(x^2 - 2x + 1)$ with the leading term of the remainder $12x - 7$, we find that the remainder is of lesser degree than the divisor

The process terminates with us dividing $12x - 7$ by $x^2 - 2x + 1$.

i.e., $\frac{2x^4+3x^2-2x+2}{(x-1)^2} = \frac{2x^4+3x^2-2x+2}{x^2-2x+1} = 2x^2 + 4x + 9 + \frac{12x-7}{x^2-2x+1}$

4. $\frac{3x^4+5x^2-7}{x^2+2x}$

$$
\begin{array}{rrrrrrrrrrrrrr}
 & & & & & & & 3x^2 & - & 6x & + & 17 & + & \frac{-34x-7}{x^2+2x} \\ \cline{4-12}
x^2 & + & 2x & \multicolumn{1}{|r}{3x^4} & + & 0x^3 & + & 5x^2 & + & 0x & - & 7 & & \\
 & & & 3x^4 & + & 6x^3 & & & & & & & & \\ \cline{4-12}
 & & & & - & 6x^3 & + & 5x^2 & + & 0x & - & 7 & & \\
 & & & & - & 6x^3 & - & 12x^2 & & & & & & \\ \cline{6-12}
 & & & & & & & 17x^2 & + & 0x & - & 7 & & \\
 & & & & & & & 17x^2 & + & 34x & & & & \\ \cline{8-12}
 & & & & & & & & - & 34x & - & 7 & & \\
 & & & & & & & & - & 34x & - & 7 & & \\ \cline{10-12}
 & & & & & & & & & & & 0 & &
\end{array}
$$

i.e., $\frac{3x^4+5x^2-7}{x^2+2x} = 3x^2 - 6x + 17 + \frac{-34x-7}{x^2+2x}$

or: $\frac{3x^4+5x^2-7}{x^2+2x} = 3x^2 - 6x + 17 - \frac{34x+7}{x^2+2x}$

5. $\frac{3x^3-6x^2+4x+5}{x^3+2x}$

$$
\begin{array}{rrrrrrrrrrrr}
 & & & & & & & & & 3 & + & \frac{-6x^2-2x+5}{x^3+2x} \\ \cline{4-10}
x^3 & + & 2x & \multicolumn{1}{|r}{3x^3} & - & 6x^2 & + & 4x & + & 5 & & \\
 & & & 3x^3 & & & + & 6x & & & & \\ \cline{4-10}
 & & & & - & 6x^2 & - & 2x & + & 5 & & \\
 & & & & - & 6x^2 & - & 2x & + & 5 & & \\ \cline{6-10}
 & & & & & & & & & 0 & &
\end{array}
$$

i.e., $\frac{3x^3-6x^2+4x+5}{x^3+2x} = 3 + \frac{-6x^2-2x+5}{x^3+2x}$

or: $\frac{3x^4+5x^2-7}{x^2+2x} = 3 - \frac{6x^2+2x-5}{x^3+2x}$